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Two Concepts of Validity and Completeness

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Abstract. A formula is (*materially*) *valid* iff all its instances are true sentences; and an axiomatic system is called (*materially*) *sound and complete* iff it proves all and only valid formulas. These are 'natural' concepts of validity and completeness, which were, however, in the course of the history of modern logic, stealthily replaced by their formal descendants: formal validity and completeness. A formula is *formally valid* iff it is true under all interpretations in all universes; and an axiomatic system is called *formally sound and complete* iff it proves all and only formulas valid in this sense. Though the step from material to formal validity and completeness may seem to be merely an unproblematic case of explication, I argue that it is not; and that mistaking the latter concepts for the former ones may lead to serious conceptual confusions.

1. Regimentation and its completeness

To start with, let us summarize some obvious facts about common logical calculi. The passage from a natural language sentence, such as

(1) *Mickey is a mouse and Donald is a duck*

to the corresponding formula of such a calculus, e.g.

(1') $P_1(T_1) \wedge P_2(T_2)$,

can be analyzed as proceeding along two different dimensions, which we can call *regimentation* and *abstraction*. The logical vocabulary of natural language becomes regimented: disambiguated, unified and standardized – a cluster of 'improvements' for which we will use the umbrella term *idealization*. In this way, for example, the natural

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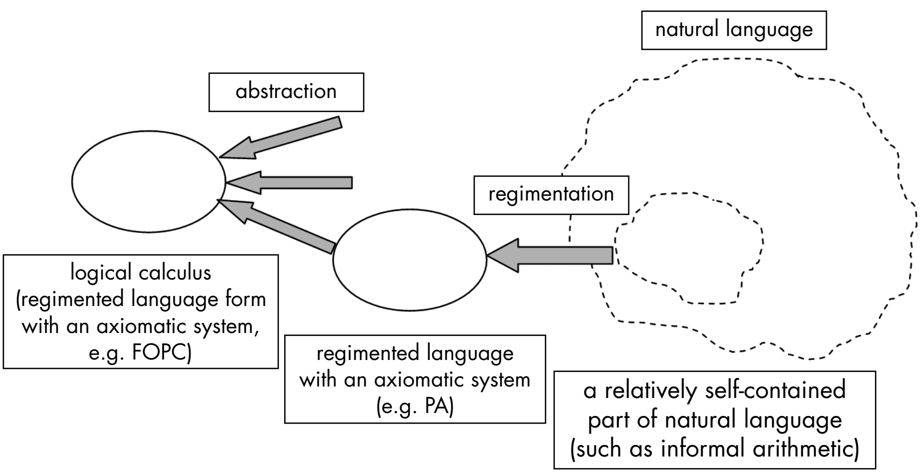
language connective "and" mutates into the operator "$\wedge$" with its precisely defined function, which largely, but not wholly, coincides with that of "and" (if only because natural language expressions do not have *precisely delimited* functions). The extralogical vocabulary becomes *abstracted* (*away*): we are not interested in concrete terms, predicates etc., for the principles we are after are those which are invariant across their variety; hence we replace them by formal parameters.

For the sake of clarity, we can decompose this step into two: first, we can imagine, we get from the natural language sentence to its regimented variant, replacing all expressions by (logical or extralogical) constants; and, second, we replace the extralogical constants by parameters, getting the *logical forms* of the original natural language statements. Hence we can imagine that in between (1) and (1') there is

(1'') $\mathbf{Mo}(\mathbf{Mi}) \wedge \mathbf{Du}(\mathbf{Do})$,

which arises out of (1) by mere regimentation and out of which (1') arises by mere abstraction.

Whereas regimentation can be seen as in principle one-to-one (which is of course not literally true, for the idealization it effects erases the distinctions between some natural language formulations), abstraction is notoriously many-to-one, it articulates the common structure of many different sentences. As an example of a merely regimented language we can consider the language of first-order Peano arithmetic (PA), whereas the language which can be seen as articulating the logical structure common to PA and any other first-order theory is that of the first-order predicate calculus (FOPC). Hence the situation can be pictured as follows:



What principles govern the process of regimentation and how can we assess whether it succeeds or fails? It seems that two conditions should be fulfilled. First, in order to be nontrivial, the regimented version of a natural language sentence must have greater perspicuity than the original – it must wear the 'logical properties' of the original that we are interested in somewhat more on its sleeve. Second, in order to count as a *regimentation* of the original, it must preserve the 'logical properties', throwing by the board only the extralogical ones.

What exactly are the 'logical properties' to be preserved and made more conspicuous, and how can they be made so? I think there is little doubt that they are basically the *inferential* properties of sentences: they consist in what each statement can be correctly inferred from and what can be correctly inferred from it. What does it mean to make these properties more conspicuous? I suggest that this means to bring the statement into a form on which we can easily base explicit formulation of inferential rules. Given this, logical form turns out to be, as Quine puts it (1980, p. 21), "what grammatical form becomes when grammar is revised so as to make for efficient general methods of exploring the interdependence of sentences in respect of their truth values."

To avoid a possible misunderstanding which may arise due to the employment of the word "inferential": I am not suggesting that natural language itself is based on *explicit* inferential rules. Some of its sentences *entail* others – and hence the latter are (*correctly*) *inferable* from the former. This notion of inferability, unlike its cousin born within the context of formal calculi, does not make for the cleft between inference and entailment. Hence I will use the terms "inference" and "entailment" largely interchangeably.

However, is there not a viable, and, as some logicians would perhaps suggest, superior, alternative to this inferential construal of 'logical properties'? Should we not say that what is to be preserved and made conspicuous is simply *meaning* – or at least some 'logical part of meaning'? Many logicians would claim that an expression's having any inferential properties cannot but be derivative to its having meaning (see esp. Prior, 1964, for a paradigmatic declaration). So should we see logic as focusing primarily on (certain aspects of) meaning[1]?

To reply to this, it is important firstly to stress that if "inference" is construed in the sense clarified above, inferential properties are definitely *semantic* properties, at least on the relevant sense of "semantics". (We should not let ourselves be misled by the fact that some logic textbooks classify even this kind of inference as a syntactic matter – the fact that it is correct to infer *Mickey is a mouse* from *Mickey is a mouse and Donald is a duck* or *Mickey is a mammal* from *Mickey is a mouse* is clearly a matter of the *semantics* of the words involved – of *and* in the former case and of *mouse* and *mammal* in the latter. This relation is 'syntacticized' only later, within logical calculi.)

Secondly, what is it to preserve meaning? The meaning of a natural language statement is a rather blurry thing, whereas if a sentence of a regimented language has anything like meaning, then it cannot but be our explicit creation, couched within set-theoretical (or similar) terms. Hence it makes little sense to imagine the relation between the former and the latter as an *identity*, it is rather what Carnap and Quine called *explication* – and given this, we face the question what makes this explication a correct one, or a good one, or an adequate one. In other words, preserving meaning cannot serve as a criterion of the

1 Tichý (1978), for example, claims that "logic is the study of logical objects (individuals, truth-values, possible worlds, propositions, classes, properties, relations, and the like) and of ways such objects can be constructed from other such objects."

adequacy of regimentation, for it is itself in need of an adequacy criterion. And it is hard to see what else could serve as an adequacy criterion save some intersentential inferential links – it is the presence of these links which is straightforwardly testable (by inspecting speakers' overt linguistic behavior)[2].

Take one of the usual ways of explicating the meaning of a sentence, namely as a set of possible worlds, common in the context of modal and intensional logics. How do we tell that the set of possible worlds assigned to a regimentation of a natural language sentence, such as (1''), really is an adequate explication of the meaning of the original sentence, such as (1)? One answer would be that this is guaranteed by the fact that we delimit the set simply as the set of those worlds in which Mickey is a mouse and Donald is a duck, i.e. in which (1) holds. But if there were nothing more to possible world semantics than this, its achievements could not but be trivial. It must, for example, hold that this set is a subset of the set assigned to *Mickey is a mouse*. Why? Because every world in which Mickey is a mouse and Donald is a duck is also a world in which Mickey is a mouse? But how do we know this? Have we visited all the worlds and seen that this is indeed the case? Surely not – it is because *Mickey is a mouse* is entailed by (1).

Hence the adequacy of a regimentation is to be measured by the preservation of the inferential structure (*modulo* the idealization underlying – and constituting the point of – the regimentation). However, as we know that as soon as we have a connective of the kind of *if … then …* of English or of the material implication of standard logic, instances of correct inference become interdefinable with necessary truths (for *B* is correctly inferable from *A* iff the sentence *if A, then B* is necessarily true; and *A* is necessarily true iff it is correctly inferable from the empty set of premises). Hence we can assume that what is to be preserved is *necessary truth*; and as we will, for the sake of simplicity, refrain from considering empirical sentences or theories, we may talk simply about *preservation of truth*. Hence the adequacy of the regimentation turns on what we will call the *material soundness and completeness* – on the extent of rendering of truth within the regimented language in terms of its formal reconstruction, typically provability, within the regimenting one[3].

Material soundness is usually not difficult to check and it appears to be a *conditio sine qua non* of a reasonable regimentation. Material completeness, on the other hand, need not be required – we may want to stay on some level of analysis which does not account for some kinds of truths. (Thus using propositional calculus as the framework

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2 This is not to say that inference is a matter of mere regularities – it is a matter of rules which presupposes not only regularity, but also the institution of a notion of *rightness* and *wrongness* (See Peregrin 2001, § 7.5).

3 The last few sentences have admittedly glossed over a number of issues which are not insubstantial. First, we assumed that what we call correct inference is always context-independent ('necessary' or 'analytic') and that we can separate it from other, context-dependent varieties of inference (kinds such as the inference of *John is in France* from *John is in Paris*). To assume this about natural languages is clearly unwarranted, but as we have already pointed out, logic is based on a purposeful idealization of natural language, which requires the replacing of fuzzy boundaries with sharp ones. Second, the precise features of *if … then …* are also fuzzy and to say that it provides for the reduction of correct inference to necessary truth might require some comments. Third, the term 'necessary' itself is not wholly clear. However, I believe that we can afford not to burden the exposition with the discussion of these themes which, from the viewpoint of the present article, are side issues.

of the regimenting language, we deliberately resign on capturing the kinds of truths which hold in virtue of the words such as *some* or *all*.)

What is essentially important is that unlike in the case of *formal* completeness, which we can encounter in logic textbooks and which we will discuss later, there is no way to decisively check for material completeness, let alone *prove* it. The reason is that the 'naturalness' of the natural language consists precisely in the fact that it is in no respect 'criterially' delimited, and hence we cannot compare the delimitation of the provability within our regimented language with the truth within natural language. What we do is precisely a 'criterial reconstruction' of a natural, and hence 'uncriterial' notion (see Peregrin, 1995).

2. Abstraction and validity

Let us now turn our attention to the step of logical analysis which follows regimentation and which leads from (regimented) sentences to their forms (embodied in *formulas* – sentences with free *parameters*). This step may appear to be almost mechanical: after we set up a boundary between that part of the vocabulary of the regimented, and consequently the regimenting, language that will be abstracted away and that which will survive (hence the 'extralogical' and the 'logical' words), we simply replace the former with parameters.

Now, of course, we also have to switch from truth to *validity*: a formula is, in itself, neither true, nor false – it can only be *valid*, i.e. true for all values of its parameters. Hence we can say that a formula is *materially valid* iff all its instances are true. (Which instances? Primarily the regimented ones, but via them also the corresponding unregimented natural language sentences.) A valid form is supposed to represent a form of 'inherently' true sentences, i.e. of sentences which are valid in force of having this form. In our case, where the form is a *logical* one, it can represent the form of *logically true* sentences. It follows that logical truth is tied to the truth of all instances of the logical form.

This approach to logical truth was foreshadowed by Bernard Bolzano's (1837) approach to analyticity; but Bolzano himself realized its serious shortcoming: it makes analyticity (and in our case logical truth) depend on the contingent fact of the richness of the language in question (a form valid according to this definition might become invalid by the introduction of a new expression enabling the articulation of a counterexample). We can call this the problem of the (possible) "poverty of language". Bolzano avoided it by basing his definition on an 'ideal' language, language *per se*, which as such cannot lack anything.

Bolzano's modem successors, especially Alfred Tarski (from Tarski, 1936, to Tarski and Vaught, 1957) avoided this recourse to an ideal language and coped with the problem in a different way. Remember that a formula is valid iff it has only true instances; in other words if each its 'instantiation', effected by a replacement of its parameters by constants, yields a true sentence. The point of replacing the meaningless parameters of a formula by meaningful constants is in gaining a sentence (composed of meaningful words); and clearly the same thing could be effected by attaching the relevant meanings directly to the parameters, without the roundabout through constants. Hence we could replace the instantiations of the formula by its *interpretations*: assignments of appropriate kinds of objects to its parameters as their denotations. This has the advantage that we solve the "poverty of language" problem without having to presuppose some such entity as an ideal language *per se*.

However, we have already pointed out that the meaning of a natural language expression is rather elusive and it may be difficult to get a grip on it – so is the Tarskian way not too thorny? The hope of Tarski and other semantically-minded logicians (notably Frege before him, and many others after him) was that what is relevant for logic is perhaps only some such feature or aspect of meanings which is sizeable in some explicit and relatively simple way. Frege, for example, hoped that the only semantic feature of a sentence which should interest a logician is its truth/falsity; and hence explicated its meaning (*Bedeutung*) as its truth value.

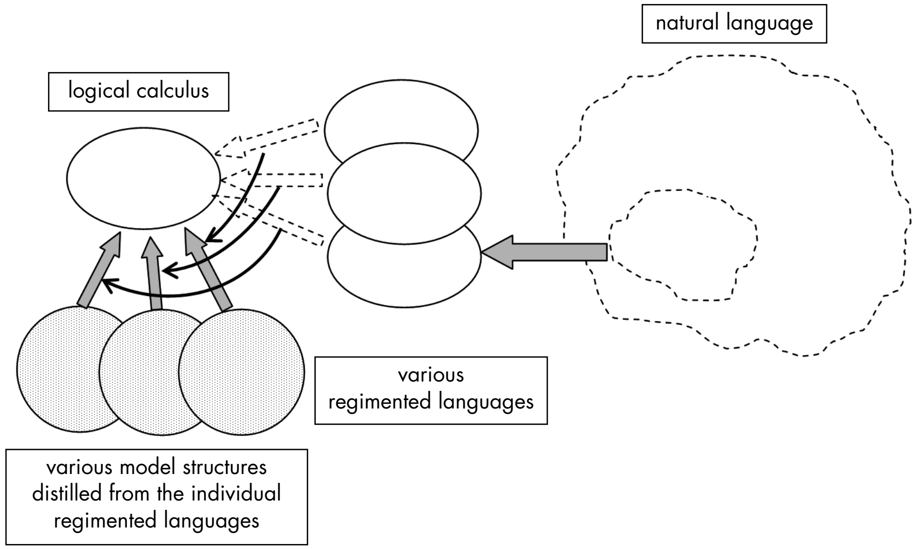
We must keep in mind that the ensuing explication of meaning (we will use the term *denotation* for the corresponding *explicatum*) is deliberately reductive. Nobody (surely not Frege) would want to claim that all true sentences have the same meaning in the intuitive sense of "meaning". Moreover, this kind of explication resembles the digitalization of an analogue signal: the semantic space in which expressions are located and which often resembles a kind of continuum is divided into discrete slots, which can then be treated as separate values to be assigned to expressions.

Given this, we need not be interested in the sentences displaying the form embodied by a formula, but only by the possible denotations of the parameters of the formula. Moreover, it is often assumed that every interpretation we must consider is uniquely determined in some finite way – typically by an assignment of values to elementary expressions and by recursively applicable rules extending the denotations from components to compounds. (This, of course, is the much discussed requirement of *compositionality* of denotations – see Werning et al., 2005.) This makes the space of interpretations particularly perspicuous and it enables us to be sure that we take all of them into account with no omissions.

Hence we can say that a formula is valid iff it is rendered true (or *satisfied*) by every possible interpretation (denotation-assignment). The problem of the "poverty of language" is solved by the successful 'digitalization' of semantics, which makes it possible to consider all denotations, irrespectively of whether they are or are not denoted by expressions of a language.

So provided our digitalization is indeed successful (in the sense that it incorporates all the semantic features of expressions to which our logical constants, now including quantifiers, are sensitive), we can replace any instance of a logical form by an interpretation which verifies the form just in case the instance is true; and moreover, we may have interpretations for which the corresponding instances are lacking because of restrictions of the expressive means of the language in question.

The following diagram illustrates what happened: the links of the previous diagram, connecting the language form, such as the FOPC, with its various instances (such as PA) mutate into the links which now connect the language form with various model structures; as indicated by the thin bold arrows:



Besides the problem of validity brought about by the process of abstraction, logical calculi also inherit the problem of soundness and completeness introduced by the process of regimentation. These concepts, however, change: a logical calculus is materially sound and complete if it proves all and only such formulas which are materially valid. The formal counterparts of these concepts then do not match provability with the material, but rather with the formal counterpart: the calculus is formally sound and complete iff it proves all and only formally valid formulas.

In this way the process of the transformation of material validity and material soundness & completeness into their formal successors appears to be successfully completed. Now, it would seem, we may forget about the former and acquiesce in the latter. But may we indeed? Is it merely another unproblematic case of explication – of a desirable, gradual movement from a fuzzy and loosely delimited concept to a clearer, sharply delimited one?

The trouble is that the existence of a gap between an *explicatum* and its *explicandum* is a fact which, though it can often be disregarded in taking the former directly as a proxy for the latter, can never be entirely *erased*. And taking its existence into account is crucial for the ability of assessing the adequacy of the explication. Otherwise we are in continual danger of letting the *explicatum* play the role of the *explicandum* even in contexts where this is impossible, i.e. where we rely on some properties of the *explicandum* which get abstracted away within the process of explication. Moreover, we may lose the ability to correct errors or infidelities possibly committed during the process of explication – cases where we abstracted away something that was relevant. And in the extreme case, this may even lead to 'arguments' that the *explicandum* 'in fact' (= as 'proved' by the explication) does not have the properties at all.

Elsewhere (Peregrin, 2001, Chapter 9) I portrayed the situation on the background of distinguishing between the 'realm of the natural' (i.e. of those entities which we *encounter* and which we can know only *empirically* and hence always *uncertainly* and *incompletely*), and the 'realm of the formal' (i.e. of those which we *define* and hence can know *non-empirically*, *certainly* and *completely*). Explication is a matter of bridging the gap between the two: of offering an item from the realm of the formal as a handy proxy for an item from the realm of the natural. The adequacy of the explication is a matter of the faithfulness of the proxy. And it is an illusion to think that we could explicate the relation of explication *itself* and thus get rid of the adequacy problem – for to be explicated means to be brought into the realm of the formal, which inevitably *creates* a problem of adequacy.

Let us make a historical remark: it was only the 'digitalization', and consequent formalization, of semantics described in this section that retroactively explicitly vindicated the *de facto* existing alliance between the logicians of the Frege-Peano-Russell school and those of the Boole-Peirce-Schröder school (see, e.g. Peckhaus, 2004). The former school was in pursuit of general linguistic forms underlying the most basic general truths and inferential patterns needed for our enterprise of proving, justifying and reasoning; the latter was after certain structural properties of sets of individuals. To exaggerate, we may say that while the former pursued generally valid linguistic forms, the latter engaged in a kind of set theory. What makes the partisans of these two quite different enterprises allies?

After the formalization of semantics took place, we can say that their alliance lies in the fact that investigating the general validity of formulas can be explicated as studying the regularities of the universes from which the constituents of the formulas draw their denotations. Of course, this was intuitively taken for granted by the adherents of both the camps much earlier than it was explicitly articulated and this was why they all felt they belonged under the common umbrella of "logic". However, in this paper we want to point out that the stealthy replacement of the concepts of material validity and completeness by their formal simulacra should not stay unnoticed.

3. The limits of formalization

What exactly must we presuppose to be able to take the proof of formal soundness & completeness as proving also the material one? As we saw, we must take for granted that the 'digitalization' of semantics underlying the Tarskian model theory is adequate in the sense that the 'digitalized' denotation of every complex expression depends only on the denotations of its parts; and that these values can be taken as responsible for the logical (i.e. inferential) properties of statements.

But is the assumption of compositionality of denotations always reasonable? For example, can we assume that truth values, which act as denotations within classical logic, are compositional? Surely not in general: it is obvious that the truth values of a sentence such as *A because B* (*The streets are wet because it rains*) is *not* uniquely determined by those of *A* and *B*. Frege obviously believed that there is some important 'core' of language where such truth-functionality *can* be assumed; and this assumption has been accepted by the mainstream of his followers. Hence what is nowadays taken to be 'standard' logic is truth-functional (in the case of propositional calculus literally; in the case of predicate, with some provisos).

How do we know that the operators of standard logic are truth-functional? Of course, because we have *defined* them to be such. This makes it possible to prove the formal completeness of standard logic: due to the guaranteed truth-functionality of the operators we have a sharply delimited range of manageable interpretations and hence it is uniquely determined when a formula is valid – and we can investigate in how far the set of valid formulas coincide with the set of theorems of some calculus.

Are the new concepts of soundness and completeness a natural explication of the concepts of material soundness and completeness dealt with above? Can the talk about these formal concepts legitimately replace the talk about the original, natural ones? Well, it is clear that this general adequacy depends on the adequacy of the explication of logical operators, which, we saw, were explicated as insensitive to any other feature of meaning save those resulting from the underlying 'digitalization' (in the particular case of standard logic as truth-functional). Can this adequacy be simply taken for granted? This is largely dubious – material implication is by no means a faithful rendering of anything within natural language; and the situation is similar with ordinary negation (if only because natural language contains no single element which could be reasonably held for the counterpart of logical negation – negating in natural language is a very complicated phenomenon which takes various guises).

This is even more true of the surplus operators of predicate logic – there is nothing in natural language which straightforwardly corresponds to either of the quantifiers. Moreover, here we also need a suitable 'digitalization' of the meanings of terms and predicates. The standard method is to take terms to denote individuals (which is not very restrictive, for an individual can be anything) and predicates are taken to denote sets of individuals (which is much more restrictive). Moreover, we also need to stipulate which sets of individuals are available (only some well-behaved – e.g. finitely specifiable ones? Or *all* of them? But what does *all sets* exactly mean in the infinite case?).

Hence the proof of formal completeness builds on the assumption of the truth-functionality of the operators; and if it is to be related to the material completeness of the calculus, we must presuppose that the truth-functional operators constitute a reasonable explication of the corresponding expressions of natural language. But it is not easy to say what exactly the quantifiers explicate. Logical analysts of language, from Russell (1905) to Quine (1980), made it plain that the relationship between quantifiers and natural language structures is very complex and that consequently the logical analysis of language is an extremely nontrivial enterprise. It seems that the ambitions of FOPC w.r.t. natural language are rather holistic: we take it that the quantifiers suffice to regiment – sometimes in a rather intricate way – a great deal of locutions of natural language. Our reasons for believing this are mostly 'inductive' – the practitioners of logical analysis usually find some way to regiment a given locution. This assumption has not been – and cannot be – *proved*, which makes it impossible to consider the proof of the formal completeness as emulating that of material completeness.

But, someone might object, why worry about natural language at all? We are replacing its cumbersome means by the streamlined means of a formal language. Why not simply leave the original means behind and never look back? Well, though there are situations where scientists, especially mathematicians, resort to a language of the form of FOPC, if logic is to be after the generally valid structures of discourse, or at least scientific discourse, then there is no way to neglect everything which is not couched in FOPC – it is precisely these theories which logic was called upon to study. Hence the question of the relationship of the formal calculus to natural language remains vital[4].

4. Hilbert & Ackerman on completeness

It may be helpful to consider some historical examples. In their *Grundzüge der theoretischen Logic*, Hilbert and Ackermann (1928) write:

> Die Vollständigkeit eines Axiomensystems lässt sich in zweierlei Weise definieren. Einmal kann man darunter verstehen, dass sich aus dem Axiomensystem alle richtigen Formeln eines gewissen, inhaltlich zu charakterisierenden Gebietes gewinnen lassen. Man kann aber auch den Begriff der Vollständigkeit schärfer fassen, so dass ein Axiomensystem nur dann vollständig heisst, wenn durch die Hinzufügung einer bisher nicht ableitbaren Formel zu dem System der Grundformeln stets ein Widerspruch entsteht.[5]

It would seem that at least the first of these two senses of "completeness" is our material completeness – it amounts to capturing all the truths of some antecedently given range. What Hilbert and Ackerman call the "stricter" delimitation of completeness is then based on the assumption that a theory complete in this sense is a theory which does not allow for a consistent extension. And as consistency of a set of statements amounts to the fact that no contradiction is inferable from the set, T is proclaimed consistent (CON(T)) iff there is no A such that $T \vdash A$ and $T \vdash \neg A$; and it is proclaimed complete iff there is no T^* such that CON(T^*) and $T \subsetneq T^*$. This is not, of course, the Tarskian explication in terms of models, but it is, nevertheless, a delimitation which allows Hilbert and Ackerman to *prove*, in the distinctively mathematical manner, that the propositional calculus is complete. Hence it must amount to a *formal* completeness.

Where did Hilbert and Ackerman move from the material to formal completeness? Was it on the way from their original to their stricter concept? Or did the original, despite appearances, already amount to formal completeness? The crucial question is how we construe their term "inhaltlich zu charakterisierenden Gebietes". What seems to come naturally is to understand "inhaltlich" as "informal" and "Gebiet" as a range of human knowledge and consequently of sentences articulating it, such as arithmetic. However, it is not easy to reconcile this reading with other things which the authors say.

4 Moreover, I do not think that we should embrace the idea of some of the pioneers of logical analysis in that natural language is inherently imperfect and that replacing it by an artificially streamlined language of logic would be a step forward. On the contrary, I suspect that the millennia of development of natural language under the pressure of natural selection have been perfecting it as a tool of communication, and that even the features which have plagued the logical analysts, such as the all-pervasive context dependence, have an important role to play.

5 "The completeness of a system of axioms may be defined in two ways. In the first case it is possible to construe it in such a way that the system allows for the establishment of all correct formulas of an informally characterized range. However, one can also grasp the concept of completeness in a stricter way, so that a system of axioms is complete only when the addition of a so far non-inferable formula to the system of axioms leads to a contradiction."

From the context of the book it is beyond doubt that the term “Formel” refers to a formula in our sense, i.e. to a statement form, containing parameters, of the kind of our (1'). However, it seems that the talk of a range of human knowledge would require reading the term “Formel” as merely a regimented statement, consisting of constants, like (1") above – it seems that when talking about a specific range we have in mind some truths specific to it, not *logical* truths, which are common to all ranges.

Moreover, Hilbert and Ackerman continue:

> Die Vollständigkeit in erstem Sinne würde hier besagen, dass man aus den Axiomen … aller immer richtige Aussagenformeln ableiten kann[6].

where the “immer richtige” formulas are just tautologies – formulas which are true independently of the truth values of their component formulas. This would indicate that this kind of completeness is already the *formal* completeness.

I think that what is going on here is a tacit fluctuation between the two different notions of completeness we separated above – between a calculus being complete in the sense of capturing all those forms of the given range that have only true instances, and completeness in the sense of capturing all formally defined tautologies. Introducing the logical connectives, the authors instruct the readers to read $A \rightarrow B$ as “if *A*, then *B*”. This totally obscures the fact that we should ask in how far we are substantiated in replacing the English “if *A*, then *B*” by $A \rightarrow B$. (And even if this were an excessive pedantry in the case of $\rightarrow$, in the case of quantifiers, it is definitely not.)

It is clear that once we replace “if *A*, then *B*” by $A \rightarrow B$ (and similarly for other logical connectives), completeness is forthcoming; it is in fact trivial. The crucial step from the material to the formal is made by the replacement; and taking this step for so unproblematic as Hilbert & Ackerman do means to conceal the very existence of the gap between the material and the formal notions. This creates the illusion that what the authors proved was *material* completeness; but we can see that this is possible only thanks to the fact that they simply identified the natural language connectives with the artificial truth-functional ones.

5. "Representational semantics" instead of model theory?

Georg Kreisel (1967) starts his texts included in the renown compilation of Hintikka with distinguishing between *formal* and *informal rigour*, the former concerning the set up of an artificial rule-based system, the latter the relationship of such system to pre-formal concepts. Hence it seems that Kreisel urges precisely what was pointed out above: that it is one thing to study relationships internal to a formal system and another things to study the relationship of this system to the preformal reality it was devised to capture.

Indeed Kreisel addresses, among others, the very concept of “intuitive logical validity” and that of the “truth in all set-theoretic structures” – hence he would seem to be addressing the central theme of this paper. However, what he says, from the viewpoint

6 “Here the completeness in the first sense would say that it is possible to infer all the always correct propositional formulas from the axioms.”

entertained here, is rather embarrassing: the only distinction between the two kinds of validity, according to him, is that a formula is intuitively logically valid iff it is true in all structures whatsoever, whereas it is true in all set-theoretic structures iff it is true "in all structures in the cumulative hierarchy". The confrontation of what seemed to be intuitive and formal validity turns out to be the confrontation merely of two versions of the model-theoretical explication of validity.

I think we have two options: either we read Kreisel as talking about a specific kind of language which is from the beginning devised to talk about set-theoretical structures, in which case it is unclear what he means by "intuitive" (for what could be meant by this word would have to be merely an intended interpretation); or we construe him as addressing the truly intuitive logical validity, i.e. truth in force of logical vocabulary alone, in which case his replacement of this concept with the truth in all structures would seem unwarranted.

This problem was noted by Etchemendy (1990, 147–8), who saw the deficiency of Kreisel exposition quite clearly:

> The problem is that Kreisel simply identifies, without argument, the intuitive notion with the model-theoretic notion of truth in all structures. … What [Kreisel's argument] shows is that, in the first-order case, truth in all structures is equivalent to truth in a restricted collection of structures. … What Kreisel's argument does not show, however, is that this extension coincides with the set of logical truths of any given first-order language – say, the logical truths of the language of elementary arithmetic.

Unfortunately, the remedy Etchemendy proposes is also not sufficient. He proposes to replace the Tarskian explication of consequence by means of one based on "representational semantics" – a theory which Etchemendy declares to be "superficially close" to Tarski's model theory, but differing in that the model structures are required to represent the possible states of the world.

The trouble is that what Etchemendy proposes instead of filling the gap within Kreisel's proposal opens up an additional one. Instead of concluding that the step from the intuitive, material concept of validity to the model-theoretic one involves some substantial and arbitrary commitments (which may, in some contexts, be quite in order, *if we are clear about them*), he tries to save the situation by means of adding *one more* commitment, namely that model structures represent possible states of affairs. And it is a commitment even more problematic than the previous ones.

As I argued at length elsewhere (see Peregrin, 1995, § 4.7), it makes little sense to imagine the space of model structures as constituted by modeling, one by one, all the possible states of the world; rather what is constitutive to this space is logical truth. The fact that we have no structure in which an individual has and at the same time does not have a property does not follow from the fact that during our mental journey through the possible states of the world we never encountered anything like this, but rather it follows from the fact that we use our language in such a way that the truth of a statement of the form *X is P and is not P* would not make sense. Possible worlds arise out of the explication of logical truth, not vice versa.

Arguments in favor of the model-theoretic approach to validity similar to those of Etchemendy were given by Priest (1999). He argues, in effect, that the model-theoretic account of consequence and validity is correct to the extent to which the model structures taken into account represent possible situations. However, unlike Etchemendy Priest does see the problem as one of failing to tackle the question of checking the 'representational adequacy', i.e. of inspecting the relationship between the model structures on which the formal application is based and the states-of-the-worlds which are alluded to by the intuitive conception. (And that this is not a trivial problem can be indicated with the help of even the simplest of English sentences, such as "It rains" – for can we really see the part or an aspect of the world which is relevant for the truth/falsity of the sentence as a first- (or, for that matter, higher-) order structure, i.e. as a couple of relations distributed among discrete individuals?)

Priest (*ibid.*, p. 189) thus concludes his model-theoretical account for validity with the following disclaimer:

> Strictly speaking, then, we have not given a final account of what it is for an inference to be valid; we have reduced the matter to that of the truth of a certain mathematical sentence. We may well ask the question of what it is for such a statement – or any mathematical statement – to be true. This is a profound question, but is far too hard to address here. One problem at a time!

However, even if we do disregard the general question of mathematical truth (which I agree with Priest is better left for another occasion), we *must* face a more down-to-earth question, namely what would make an answer to the mathematical question helpful for answering the original one? And this question is inevitable, because if the answer is *nothing*, then wrestling with the mathematical question would be a waste of time.

To repeat: I do think that dealing with the mathematized version of the problem *is* helpful; but I also think that silence about the question why and how it is helpful is precarious.

6. Validity of arguments

Finally, let us consider an example of a contemporary logic textbook: I take one which I hold for one of the very best, namely *Logic, language and meaning* by the virtual L.T. F. Gamut[7]. In the introduction of the book, the authors, delineating the subject matter of logic, claim that logic addresses argument schemata, i.e. schemata which arise from taking real arguments and abstracting away some of its component (those which we take to be 'extralogical'). Hence an argument schema (precisely in the way outlined in the beginning of this paper) is a type whose tokens arise out of replacement of the parameters of the schema by certain concrete expressions. The authors give the example

7 A collective pseudonym of the Dutch logicians J. van Benthem, J. Groenendijk, M. Stokhof, D. de Jong and H. Verkuyl.

(11)
$$\begin{array}{c} A \text{ or } B \\ \underline{\text{Not } A} \\ B \end{array}$$

and claim:

> The letters A and B stand for arbitrary sentences. Filling in actual sentences for them, we obtain an actual argument. Any such substitution into schema (11) results in a valid argument, which is why (11) is said to be a *valid argument schema*.

This leads them to the following conclusion w.r.t. the subject matter of logic:

> Logic, as the science of reasoning, investigates the validity of arguments by investigating the validity of argument schemata. For argument schemata are abstractions which remove all those elements of concrete arguments which have no bearing on their validity. As we have seen, argument schemata can be formed from a variety of expressions and syntactic constructions. Usually they are not all considered together but are taken in groups. So, for example, we can concentrate on those argument schemata which can be formed solely from sentences, grammatical conjunctions, like *or* and *if … then*, and negation. Or we can single out arguments containing quantifying expressions.

I think that little can be objected to this; but surely the reader would expect that after the authors introduce the machinery of modern formal logic, they would return to this and show how the new machinery can be put into the services of characterizing valid arguments. Armed with the apparatus of standard logic, they indeed return to the topic in Chapter 4, where they give a modified explanation of the concept of validity of an argument:

> Translating the assumptions of a given argument into predicate logic as the sentences $\phi_1, \ldots, \phi_n$ and its conclusion as the sentence ψ, we obtain an argument schema $\phi_1, \ldots, \phi_n / \psi$. It has $\phi_1, \ldots, \phi_n$ as its premises and ψ as its conclusion. If accepting $\phi_1, \ldots, \phi_n$, commits one to accepting ψ, then this argument schema is said to be valid, and ψ, is said to be a logical consequence of $\phi_1, \ldots, \phi_n$. An informal argument is also said to be valid if it can be translated into a valid argument schema.

Now this starts to be slightly confusing. As $\phi_1, \ldots, \phi_n / \psi$ is said to be an argument *schema*, the "translation" mentioned must replace extralogical symbols by what are, in effect, *parameters* and hence must amount to what we called *abstraction* above. However, what then does it mean that "accepting $\phi_1, \ldots, \phi_n$, commits one to accepting ψ"? As it seems hard to construe 'accepting a scheme' otherwise than as a shorthand for accepting all instances of the scheme, we presume that what the authors claim is that accepting *any instance* of $\phi_1, \ldots, \phi_n$ commits one to accepting the *corresponding instance* of ψ. But what instances are to be considered here?

From what the authors claimed at the beginning of the book, it would seem that we are to consider the *natural language* instances – those from which the scheme was originally abstracted (in the authors' terminology, those which are "translated" by the schema). Only thus is this definition of validity in accordance with the one given at the beginning of the book. However, then it is not clear what the connection is between this pre-formal notion and the formal notions of semantic and syntactic validity to which the authors restrict their attention from that point on. Are the formal notions explications of the pre-formal one? Obviously, they are meant to be, but no argument whatsoever is given for the claim that they are.

Or should we consider the instances referred to as instances within the language of FOPC? In such a case, the semantic and syntactic validity would clearly capture this notion of validity (the syntactic more, while the semantic one less) directly, but as a consequence it would be a formal concept, not obviously related to the preformal one. Again, a discussion of the relationship between the former and the latter is missing. Although it is often legitimate to work with various kinds of simplifications, idealizations and explications, the replacement of the explicandum with the explicans should not go without saying.

7. The 'digitalization of thought'

The often unreflected step from material validity and completeness to their formal counterparts couched in terms of the standard, Tarskian semantics consists in the 'digitalization' which renders any simple contentful utterance as a statement of a relationship between objects. This, of course, is very natural; but the ventures of those who have tried to develop a picture of the world based exclusively on this kind of 'object-based ontology' (*viz.* Russell, 1914, or Wittgenstein, 1922; or later Barwise and Perry, 1983) indicate that it is not problem free.

In an unpublished dissertation, John MacFarlane (ms.) distinguishes three concepts of formality which usually get conflated during discussions about the formality of logic. First, formality may amount to constitutivity w.r.t. ('giving form to') thought as such. Second, formality may amount to neutrality to kinds of objects. Third, formality can mean abstraction of all kinds of meaning. Let us call these three concepts formality$_1$, formality$_2$ and formality$_3$.

MacFarlane points out that while it was mostly formality1 that was historically seen as the distinctive feature of logic, in the twentieth century many logicians and philosophers of logic have proposed the delimitation of logic based on formality$_2$. (I think this idea originated with Tarski, 1986; recently it has been elaborated by several authors – see, e.g., Sher, 1991.) And as MacFarlane duly points out, it is strange that the proponents of this notion of formality of logic do not discuss its relationship to the more traditional one.

In fact the step from formality$_1$ to formality$_2$ is in some respects parallel to the step from material to formal validity – in particular it presupposes the 'digitalization' of thought. To assume that formality1 is nothing over and above formality$_2$ (or that the latter is a straightforward 'explication' of the former) is to assume that any thought is a thought essentially about objects – or, expressed in a post-linguistic-term idiom, that every significant statement is a statement about an object or objects. I do not think this is viable. (Consider the example already mentioned: the sentence “It rains” is not a claim about an object or objects – at least if we do not stretch the concept of object to the point of vacuousness.)

In addition, it presupposes something *more*, namely that the semantic content of all expressions which are not directly means of referring to objects (esp. the logical ones) are sensitive to nothing more than to the identity of the objects referred to by the basic ones – in other words that the ‘digitalization’ underlying the ‘object-based ontology’ is adequate. And this is something which is far from obvious.

8. Conclusion

The concepts of material validity and completeness are different from those of formal validity and completeness. While the former amount to the confrontation of a formal system with its preformal prototype, the latter is a matter internal to the formal system. And whereas it is often useful and legitimate to replace an informal entity by its formal explication, the attempt to formally explicate the very relationship between an explicatum and an explicandum portends a vicious circle. I am afraid that many writers on logic overlook this danger; and I have tried to indicate what kinds of confusions this may give rise to.

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