The version of this paper printed in the book contains errors (which resulted from re-editing of the text after I did the proofs). Please use this correct version instead of it.

Chapter 3

SEMANTICS AS BASED ON INFERENCE

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1. There is more to semantics than inference ...

We may say that logic is the study of consequence; and the pioneers of modern formal logic (especially Hilbert, but also, e.g., the early Carnap) hoped to be able to theoretically reconstruct consequence in terms of the relation of *derivability* (and, consequently, necessary truth in terms of *provability* or *theoremhood* – derivability from an empty set of premises). The idea was that the general logical machinery will yield us derivability as the facsimile of the relation of consequence, and once we are able to formulate appropriate axioms of a scientific discipline, the class of resulting theorems will be the facsimile of the class of truths of the discipline.

These hopes were largely shattered by the incompleteness proof of Gödel (1931): this result appeared to indicate that there was no hope for fine-tuning our axiom systems so that theoremhood would come to align with truth. Tarski (1936) then indicated that there are also relatively independent reasons to doubt that we might ever be able to align derivability with consequence: he argued that whereas intuitively the statement *every natural number has the property P* follows from the

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set of statements $\{n \textit{ has the property } P\}_{n=1,\dots,\infty}$, it can never be made derivable from it (unless, of course, we stretch the concept of derivability as to allow for *infinite* derivations).

These results reinforced the picture, present in the back of many logicians' minds anyway, of logic as trying to capture, using our parochial and fatally imperfect means, truth and consequence that are somewhere 'out there', wholly independent of us. And as whether a sentence is (necessarily) true[2] and what follows from it is a matter of its *meaning*, it also appeared to indicate that there must be much more to meaning than can be captured by inference rules. In particular, there must be more to the meanings of logical and mathematical constants than is captured by the inference rules we are able to construct as governing them.

Symptomatic of this state of mind is Arthur Prior's famous denial[3] of the possibility of assigning a logical constant its meaning by means of inferential rules:

> It is one thing to define 'conjunction-forming sign', and quite another to define 'and'. We may say, for example, that a conjunction-forming sign is any sign which, when placed between any pair of sentences P and Q, forms a sentence which may be inferred from P and Q together, and from which we may infer P and infer Q. Or we may say that it is a sign which is true when both P and Q are true, and otherwise false. Each of these tells us something that could be meant by saying that 'and', for instance, or '&', is a conjunction−forming sign. But neither of them tells us what is meant by 'and' or by '&' itself. Moreover, each of the above definitions implies that the sentence formed by placing a conjunction-forming sign between two other sentences already *has* a meaning. For only what already has a meaning can be true or false (...), and only what already has a meaning can be inferred from anything, or have anything inferred from it. (1964, p.191)

2. ... but there cannot be more!

Some of the most outstanding philosophers of language of the XX. century, on the other hand, arrived at the conclusion that there could be hardly any way of furnishing our words with meanings save by subordinating them to certain rules – the rules, as Wittgenstein (1953) famously put it, of our *language games*.

The point of departure of Wittgenstein's later philosophy was the recognition that seeing language, as he himself did earlier in the *Tractatus*, as a complex set of *names* is plainly unwarranted. Most of our

[2] In this paper we will have nothing to say about empirical statements and hence about other than necessary truths.
[3] See Prior (1960); and also Prior (1964).

words are not names in any reasonable sense of the word *name*, and hence if they have meaning, they must have acquired it in a way other than by having been used to christen an object. And Wittgenstein concluded that the only possible way this could have happened is that the words have come to be governed by various kinds of rules. Thus, in his conversation with the members of the Vienna Circle he claims (see Waisman (1984), p. 105):

> For Frege, the choice was as follows: either we are dealing with ink marks on paper or else these marks are signs of *something,* and what they represent is their meaning. That these alternatives are wrongly conceived is shown by the game of chess: here we are not dealing with the wooden pieces, and yet these pieces do not represent anything – in Frege's sense they have no meaning. There is still a third possibility; the signs can be used as in a game.

This indicates that for Wittgenstein, Prior's claim "only what already has a meaning can be inferred from anything, or have anything inferred from it" would be no more plausible than the claim that only what is already a pawn, a knight etc. can be subordinated to the rules of chess: just like we *make* a piece of wood (or of something else) into a pawn or a knight by choosing to treat it according to the rules of chess[4], we make a sound- or an inscription-type into a meaningful word by subordinating it to the rules of language.

As Sellars (1974) puts it, there are essentially three kinds of rules governing our words:

> *language entry transitions*, or rules of the *world-language type intralinguistic moves*, or rules of the *language-language type language departure transitions*, or rules of the *language-world type*

Whereas the first and the last type is restricted to *empirical* words, nonempirical words are left with being furnished with meaning by means of the middle one, which are essentially *inferential* rules. Meaning of such a word thus comes to be identified with its *inferential role*[5].

In some cases, this view appears to be markedly plausible (*pace* Prior[6]). How could "and" come to mean what it does? By being attached, as a

[4]Note that though the shape of the piece is usually indicative of its role, having a certain shape is neither necessary, nor sufficient to be, say, a pawn.

[5]Inferentialism in Wittgenstein's later philosophy is discussed bv Medina (2001); for an account of Sellars' semantics see Marras (1978).

[6]What Prior did show was that not every kind of inferential pattern can be reasonably taken as furnishing a sign with a meaning. (His famous example is the 'vicious' pattern A $\Rightarrow$ A tonk B; A tonk B $\Rightarrow$ B.) A But it is hard to say why it should follow that the meaning of 'and' is not determined by the obvious pattern: is not what one learns, when one learns the meaning of 'and', precisely that A and B is true (or correctly assertible) just in case both A and B are? (See Peregrin (2001), Chapter 8.)

label, to the standard truth function? But surely we were in possession of "and" long before we came to possess an explicit concept of function – hence how could we have agreed on calling it "and"? The Sellarsian answer is that by accepting the inference pattern

A *and* $B \Rightarrow A$
A *and* $B \Rightarrow B$
$A, B \Rightarrow A$ *and* B

(which need not have been, and surely was not, a matter of accepting an explicit convention, but rather of handling the signs involved in a certain way).

In other cases it is perhaps less straightforwardly plausible, but still urged by many theoreticians. How could numerals come to mean what they do? By being attached, as labels, to numbers? But how could we achieve this? Even if we submit that numbers quite unproblematically exist (within a Platonist heaven), we surely cannot point at them, so how could we have agreed on which particular number would be denoted by, say, the numeral "87634"? The inferentialist has an answer: numbers are secondary to the rules of arithmetic, such as those articulated by means of Peano axioms; and hence 87634 is simply a 'node' within the structure articulated by the axioms, namely the node which is at a particular distance from zero. As Quine (1969), p.45 puts it: "There is no saying absolutely what the numbers are, there is only arithmetic."

All of this appears to suggest that there cannot be more to the meanings of logical and mathematical constants than is captured by the inference rules governing them. Hence we appear to face the following question: *Can the standard meanings of logical and mathematical constants be seen (pace Tarski & comp.) as creatures of entirely inferential rules?*

3. Disjunction

An inferentialist has an easy time while grappling with "and"; but troubles begin as soon as he turns his attention to the (classical) "or". There seems to be no set of inferential rules pinning down the meaning of "or" to the standard truth-function. Indeed "or" can be plausibly seen as governed by

$A \Rightarrow A$ *or* B;
$B \Rightarrow A$ *or* B,

but then we would need to stipulate that *A or B* is not true unless either *A* or *B* is. Of course we might have

not-A, *not-B* $\Rightarrow$ *not*-(*A or B*),

but this presupposes the (classical) *not*, and hence only shifts the burden of inferential delimitation from *not* to *not*, which is surely no easier.

In fact as long as we construe inference as amounting to truth–preservation, there can be no way to inferentially express that a sentence is, under some conditions, *not* true. (And it is well-known that the axioms of the classical propositional calculus admit theories with true disjunctions of false disjuncts[7]).

Perhaps the way out of this could be to part company with classical logic (and subscribe to, say, something like intuitionism) – but how, then, could the classical "or" have come into being?

I envisage two kinds of answers to this question:

(1) It did not exist prior to our having an explicit idea of function and was only later procured from the pre-classical "or" by means of an explicit rectification.

(2) There is a stronger (and still reasonable) concept of inferential pattern such that there *is* an inferential pattern able to grant "or" its *classical* semantics.

In this paper I aim to explore the second alternative. The proposal I will make is that an inferential pattern should be read not simply as giving a list of (schematic) instances of inference, but rather as giving a list which is purported to be *exhaustive*. Why should it be read in this way? Because that is what we standardly mean when we make lists or enumerate. If I say "My children are Tom and Jerry", then what is normally taken for granted is that these are *all* my children. This has been noted by McCarthy (1980), whom it led to the model-theoretic concept of *circumscription*; and indeed our proposal is parallel to McCarthy's (see also Hintikka (1988) for an elaboration).

To say that we should construe inferential patterns in this way is to say that we should read them as containing, as it were, an implicit

[7]It is usually assumed that the proofs of soundness and completeness of the propositional calculus establish that its axiomatics and its truth-functional semantics are two sides of the same coin. But this is not true in the sense that the axiomatics would pin down the meanings of the connectives to the usual truth-functions. It fixes their meanings in the sense that if the meanings are truth-functions, then they are the usual ones, but it is compatible also with certain non-truth functional interpretations.

... *and nothing else*: hence

$A \Rightarrow A$ *or* B
$B \Rightarrow A$ *or* B

should be read as "A *or* B is true if either A is true, or B is true – *and in no other case*". It is clear that by this reading, the correct classical semantics for "or" is secured.

I gave a detailed discussion of how this approach fares w.r.t. individual logical connectives elsewhere (see Peregrin, in print), so here we may ascend to a more global perspective.

4. Standard properties of inference

Let us make our conceptual framework a bit more explicit. What we call *inference* is a relation between sequences of sentences and sentences – we assume that languages come with their relations of inference (which is constitutive of their semantics). An inference is called *standard* if it has the following properties (where A, B, C stand for sentences and X, Y, Z for sequences thereof):

Ref ['reflexivity']: $A \Rightarrow A$
Cut ['transitivity']: if $X \Rightarrow A$ and $YAZ \Rightarrow B$, then $YXZ \Rightarrow B$
Con ['contractibility']: if $XAYAZ \Rightarrow B$,
then $XYAZ \Rightarrow B$ and $XAYZ \Rightarrow B$
Ext ['extendability']: if $XY \Rightarrow B$, then $XAY \Rightarrow B$

Note that **Con** and **Ext** together entail

Perm ['permutability']: if $XABY \Rightarrow C$, then $XBAY \Rightarrow C$

Indeed, if $XABY \Rightarrow C$, then, by **Ext**, $XABAY \Rightarrow C$, and hence, by **Con**, $XBAY \Rightarrow C$.

An *inferential structure* is a set of sentences with an inference relation. A *standard inferential structure* is an inferential structure whose inference obeys **Ref**, **Cut**, **Con** and **Ext**. It is clear that within a standard inferential structure, inference can be construed as a relation between *sets* of sentences and sentences.

We will also consider 'more global' properties of inferential structures. In an inferential structure each sentence can have a negation, each pair of sentences can have a conjunction, disjunction etc. Using 'extremality conditions', we can characterize these standard logical junctions as follows (cf. Koslow (1992))[8]:

Conj: $A \wedge B \Rightarrow A$; $A \wedge B \Rightarrow B$; if $C \Rightarrow A$ and $C \Rightarrow B$, then $C \Rightarrow A \wedge B$
Disj: $A \Rightarrow A \vee B$; $B \Rightarrow A \vee B$; if $A \Rightarrow C$ and $B \Rightarrow C$, then $A \vee B \Rightarrow C$
Impl: $A\ A \rightarrow B \Rightarrow B$; if $A\ C \Rightarrow B$, then $C \Rightarrow A \rightarrow B$
Neg: $A\ \neg A \Rightarrow B$; if $A\ C \Rightarrow B$ (for every B), then $C \Rightarrow \neg A$

A standard inferential structure will be called *explicit*, if it has conjunctions, disjunctions, implications and negations.

If we assume exhaustivity in the sense of the previous section, there is no need to spell out the extremality conditions explicitly: $A \Rightarrow A \vee B$ and $B \Rightarrow A \vee B$ together come to mean that the disjunction is true if one of the disjuncts is, and the exhaustivity assumption yields that it is true *in no other case* – hence that it is false for both disjuncts being false. Hence, given this, we can abbreviate the above definitions to

Conj: $A \wedge B \Rightarrow A$; $A \wedge B \Rightarrow B$
Disj: $A \Rightarrow A \vee B$; $B \Rightarrow A \vee B$
Impl: $A\ A \rightarrow B \Rightarrow B$
Neg: $A\ \neg A \Rightarrow B$

5. From inferential roles to possible worlds

By *the* (*inferential*) *role of* A we will understand the specification of what A is inferable from and what can be inferred from it together with other sentences. Hence the role of A can be represented as $<A^+,A^->$, where

$$A^+ = \{X \mid X \Rightarrow A\}$$
$$A^- = \{<X_1,X_2,Y> \mid <X_1 A X_2 \Rightarrow Y>\}.$$

If the inference obeys **Cut** and **Ref**, then $A^+ = B^+$ iff $A \Leftrightarrow B$ iff $A^- = B^-$. Indeed: (1) If $A^+ = B^+$, then as $A \in A^+$ (in force of **Ref**), $A \in B^+$, and so $A \Rightarrow B$. By parity of reasoning, $B \Rightarrow A$, and hence $A \Leftrightarrow B$. (2) If $A \Leftrightarrow B$, then if $X \in A^+$ and hence $X \Rightarrow A$, it follows (by **Cut**) that X

[8]Note that here 'conjunction' does not refer to a specific sign (and similarly for the other connectives). 'Conjunction of A and B' is a sentence with certain inferential properties, and not necessarily of a certain syntactic structure (such as A and B joined by a conjunction-sign). 'Conjunction' can then be seen as a relation between pairs of sentences and sentences (not generally a function, for we may have more than one different – though logically equivalent – conjunction of A and B).

$\Rightarrow B$ and hence $X{\in}B^+$. This means that $A^+{\subseteq}B^+$. Conversely, $B^+{\subseteq}A^+$ and hence $A^+{=}B^+$. (3) If $A^-{=}B^-$, then as $<<>,<>,A>{\in}A^-$ (in force of **Ref**), $<<>,<>,A>{\in}B^-$ (where $<>$ denotes the empty sequence) and so $B \Rightarrow A$. By parity of reasoning, $A \Rightarrow B$, and hence $A \Leftrightarrow B$. (4) If $A \Leftrightarrow B$, then if $<X_1,X_2,Y>{\in}A^-$, and hence $X_1AX_2 \Rightarrow Y$, it follows (by **Cut**) that $X_1BX_2 \Rightarrow Y$ and hence $<X_1,X_2,Y>{\in}B^-$. This means that $A^-{\subseteq}B^-$. Conversely, $B^-{\subseteq}A^-$, and hence $A^-{=}B^-$.

It follows that in this sense we can reduce the inferential role of A to whichever of its halves, in particular to A^+. Moreover, if we write $Y_1{\oplus}...{\oplus}Y_\mathrm{n}$ for the set of n-tuples of strings of formulas $X_1...X_\mathrm{n}$ such that $X_1{\in}Y_1$, ..., $X_\mathrm{n}{\in}Y_\mathrm{n}$, it is the case that

$$A_1...A_\mathrm{n} \Rightarrow A \text{ iff } {A_1}^+{\oplus}...{\oplus}{A_\mathrm{n}}^+ \subseteq A^+,$$

and if inference obeys also **Con** and **Ext** (hence if the inferential structure is standard), then

$$A_1...A_\mathrm{n} \Rightarrow A \text{ iff } {A_1}^+{\cap}...{\cap}{A_\mathrm{n}}^+ \subseteq A^+.$$

Indeed: (1) If $A_1...A_\mathrm{n} \Rightarrow A$ and $X{\in}{A_1}^+{\cap}...{\cap}{A_\mathrm{n}}^+$, then, due to **Cut**, $X...X \Rightarrow A$, which, in force of **Con**, reduces to $X \Rightarrow A$ and hence to $X{\in}A^+$. (2) On the other hand, it follows by **Ref** that $A_\mathrm{i}{\in}{A_\mathrm{i}}^+$ for i=1,...,n, and it then follows by **Ext** that $A_1...A_\mathrm{n}{\in}{A_\mathrm{i}}^+$ for i=1,...,n; hence if ${A_1}^+{\cap}...{\cap}{A_\mathrm{n}}^+ \subseteq A^+$, then $A_1...A_\mathrm{n}{\in}A^+$ and hence $A_1...A_\mathrm{n} \Rightarrow A$. (See Van Benthem (1977, Chapter 7) for a more extensive exposition.)

Logical equivalence, $\Leftrightarrow$, is a congruence w.r.t. conjunctions, disjunctions, negations and implications. This means that if $A \Leftrightarrow A'$, $B \Leftrightarrow B'$, C is a conjunction of A and B, and C' is a conjunction of A' and B', then $C \Leftrightarrow C'$ (and similarly for the other connectives). Indeed: If C is a conjunction of A and B, then $C \Rightarrow A$ and $C \Rightarrow B$, and hence, in force of the fact that $A \Rightarrow A'$ and $B \Rightarrow B'$, $C \Rightarrow A'$ and $C \Rightarrow B'$. But as for every D such that $D \Rightarrow A'$ and $D \Rightarrow B'$ it is the case that then $C' \Rightarrow D$, it follows that $C' \Rightarrow C$. By parity of reasoning, $C \Rightarrow C'$; and hence $C \Leftrightarrow C'$.

This means that we can pass from an explicit standard inferential structure to what in algebra is called its *quotient*, i.e. a structure consisting of the equivalence classes (modulo $\Leftrightarrow$) of sentences with conjunctions etc. adjusted to act on them (which we know can be done precisely because $\Leftrightarrow$ is a congruence). This can be observed as passing from sentences to *propositions*, and from sentential operators to *propositional* operators[9]. It is easy to see that the quotient structure is a Boolean algebra (with the adjusted operations of conjunction, disjunction and negation playing the role of join, meet and complement, respectively). Then, in force of Stone's representation theorem[10], it can be represented as an algebra of sets of its own ultrafilters. And as its ultrafilters correspond to just the maximal consistent theories, each sentence A can be, from the viewpoint of its 'inferential potential', characterized in terms of the set of those maximal consistent theories to which it belongs.

Now these theories can be seen as descriptive of 'possible worlds' representable by the language. Moreover, if the language in question has the usual structure of that of the predicate calculus, then the theories can be used to directly produce the 'worlds' – i.e. models – by means of the well-known construction of Henkin (1950). This means that the standard possible-worlds-variety of semantics can be seen as a means of representing a certain (standard) kind of inferential structure.

6. Representing non-standard inferential structures

Non-standard inferential structures yield us, in this way, non-standard varieties of semantic representation. If we withdraw **Con** and **Ext** (which is suggested, e.g., by considering the anaphoric structure of natural language, which appears to violate **Perm**), then the inferential structure ceases to be Boolean and does not yield the standard possible world semantics.

The closest analogue of conjunction within such a setting is what is usually called *fusion* (Restall (2000)):

Fusion: if $X \Rightarrow A$ and $Y \Rightarrow B$, then $XY \Rightarrow A \circ B$;
if $X \Rightarrow A \circ B$ and $YABZ \Rightarrow C$, then $YX\mathrm{Z} \Rightarrow C$

Assuming **Cut** and **Ref**, we can prove the associativity of $\circ$: The definition yields us, via **Ref**,

[9]In fact, the inferential roles as defined here (which I called the primary role elsewhere – see Peregrin (in print)) are reasonably taken to explicate propositions only in case of inferential structures which are at most intensional (i.e. for which logical equivalence entails intersubstitutivity w.r.t. logical equivalence, in the sense that for every A, B and C, A $\Leftrightarrow$ B entails C $\Leftrightarrow$ C[A/B]). For hyperintensional languages we should consider secondary inferential roles, which are not only a matter of what A itself is inferable from and what can be inferred from it, but also of what the sentences containing A are inferable from and what can be inferred from them.

[10]See, e.g., Bell & Machover (1977, Chapter 4).

(i) $AB \Rightarrow A{\circ}B$; and
(ii) if $YABZ \Rightarrow C$, then $Y(A{\circ}B)\mathrm{Z} \Rightarrow C$.
These can then be used to prove, with the employment of **Cut**, that $A{\circ}(B{\circ}C) \Leftrightarrow ABC \Leftrightarrow (A{\circ}B){\circ}C$. Moreover, under such assumptions we can show that if C is such that $<> \Rightarrow C$ (i.e. it is a theorem), then $B{\circ}C \Leftrightarrow B \Leftrightarrow C{\circ}B$: for (i) yields $A \Rightarrow A{\circ}C$ and $A \Rightarrow C{\circ}A$, whereas (ii) yields the converse.

This means that if the structure has fusions and there exists a C of this kind (which is certainly the case if we assume some suitable 'protoclassical' versions of disjunction, implication and negation), the corresponding propositional structure (i.e. the quotient structure modulo $\Leftrightarrow$) is a *monoid*[11]. In this case, the most natural thing appears to be to represent the propositions as some kinds of functions. And indeed it turns out that the inferential potentials of the sentences A can be now represented as

$$A^{*}= \{<X,XY> \mid Y \Rightarrow A\}.$$

In this case, it follows from the results of Van Benthem (1977, Chapter 7) that (where $\bullet$ represents functional composition)

$$A_1...A_\mathrm{n} \Rightarrow A \text{ iff } A_1{}^{*}\bullet...\bullet A_\mathrm{n}{}^{*} \subseteq A^{*}$$

Indeed: (1) If $A_1...A_\mathrm{n} \Rightarrow A$, then if $<X,XY>\in A_1{}^{*}\bullet...\bullet A_\mathrm{n}{}^{*}$, then $Y = Y_1...Y_\mathrm{n}$, where $Y_\mathrm{i} \Rightarrow A_\mathrm{i}$, and hence (due to **Cut**) $Y_1...Y_\mathrm{n} \Rightarrow A$; hence $<X,XY_1...Y_\mathrm{n}>\in A^{*}$. (2) If, on the other hand $A_1{}^{*}\bullet...\bullet A_\mathrm{n}{}^{*} \subseteq A^{*}$, then, due to **Ref**, $<X, XA_1...A_\mathrm{n}>\in A^{*}$ for every X, which means that $A_1...A_\mathrm{n} \Rightarrow A$.

Thus, in this way inferential roles yield one of the common varieties of dynamic semantics based on the so-called *updates*[12].

7. Consequence via inference

All of this apparently suggests that we can construe the common creatures of formal semantics, such as intensions or updates, as 'encapsulated inferential roles'. However this yields us straightforwardly always the Henkin semantics, not the standard one – and hence also never the Tarskian 'second-order' consequence. (Thus, the inferential structure of

[11] That dynamic semantic is "monoidal semantics" is urged by Visser (1997).
[12] Van Benthem (1977, Chapter 7) also discusses other varieties of dynamic semantics corresponding to other 'subclassical' sets of assumptions about inference.

Peano arithmetic yields us more than one 'possible world' [= model], which blocks *every natural number has the property P* being the consequence of $\{n\ has\ P\}_{n=1,\ldots,\infty}$). However, if we admit that 'enumerative' inferential patterns, such as those governing the expressions of Peano arithmetic, incorporate implicit exhaustivity assumptions (in the very way inferential patterns characterizing logical operators do) and hence involve extremality (in the sense of Hintikka (1989)), we can see inferential roles as yielding even the standard semantics and 'second-order' consequence.

Indeed: look at the Peano axioms as a means of enumeration of natural numbers (and there is little doubt that this was their original aim). What they say is that zero (or one, which was their original starting point) is a number, and the successor of a number is always again a number. This yields us the standard natural numbers, but cannot block the occurrence of the non-standard ones after them. However, if we add *and nothing else is a number*, we cut the number sequence down to size: only those numbers which are *needed* to do justice to the Peano axioms are admitted; the rest are discharged.

To avoid misunderstanding, I do not think that the exhaustivity assumption can be somehow directly incorporated into logic to yield us a miraculous system which would be both complete and have the standard semantics – this, of course, would be a sheer daydream. If we admit that our inferential patterns do contain the implicit exhaustivity assumption, we must condone the fact that therefore the patterns cease to be directly turnable into proof-procedures. My point here was that we *can* get semantics, even the 'most semantical one', out of something which can still reasonably be seen as inferential patterns; and thus we vindicate the Wittgensteino-Sellarsian claim that what our words mean cannot ultimately rest solely on the rules we subordinate them to.

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