‘Fregean’ Logic and ‘Russellian’ Logic

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Summary. I do not think there is one true answer to the question *What is logic?*. There are, clearly, good and less good answers, and there are answers which are plainly wrong; but the term ‘logic’ has been employed, throughout the history of the subject matter, in such diverse ways that no single one of the uses can be said to be *the* correct one. However, even among the answers which are acceptable on historico-semantical grounds there are still, without doubt, good and less good ones, in the sense of more and less useful. In this paper, I will argue for a certain, rather narrow conception of logic; and I am going to argue that it is not only an acceptable answer, but also one which is more useful and fruitful than its alternatives. I will argue that when setting the agenda for logic we must keep ourselves grounded; for, as I will try to indicate, it was precisely a down-to-earth conception of logic which underlay the jump start into the era of modern symbolic logic that occurred in the late nineteenth century, notably within the work of Gottlob Frege. I will compare his notion of logic with some rival ones and aim to show that the alternatives are either wrong or unmanageable.

1. Frege’s *Begriffsschrift*

Philosophers have always dreamt of a language which would be more suitable for the purpose of solving their problems (or maybe even all problems) than the ordinary language with which God or Nature equipped us. Many of them suspected that philosophical problems arise partly or wholly due to the fact that our ordinary language does not allow us to express our ideas and thoughts precisely enough – and that all would improve if we had a language which would be in perfect accordance with our thinking and/or our world. According to this view, the basic requirement of a philosopher (or also of a scientist) is a kind of ‘alphabet of human ideas’ which would enable them ‘through the connection of its letters and the analysis of the words, which consist of them, to discover and to assess everything else’ (Leibniz). However, the creation of such an alphabet presupposes a small ‘detail’: it is necessary to collect and classify all our thoughts and ideas, to find out which of them are not adequately expressed in our language and which are expressed somehow distortedly, and to clear away these shortcomings. The problem is, of course, that nobody had any idea how to achieve this, *viz* how to get hold of ‘ideas’ by-passing words expressing them. Thus, Leibniz’s project of a *calculus ratiocinator*, analogously with similar proposals, has remained a mere utopian ideal.

Nevertheless, today we do have something at least partly resembling Leibniz’s ‘calculus of rationality’: we have the symbolic languages developed by logicians. True, they are far cry from a tool facilitating the immediate solution or dissolution of philosophical problems; it is, however, undeniable that in the case of many philosophical (and scientific) problems they have helped us if not solve them, then at least to make them more perspicuous or to gain new, helpful

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insights into them[1]. What has lead us, philosophers of the twentieth century, to this partial success – when so many of our predecessors failed?

I think that it is now generally accepted that the man to whom we are most indebted is Gottlob Frege, and I am going to argue that what is especially important in this respect is what I would like to call Frege's *down-to-earthness*. The point is that when Frege launched his investigations, which resulted in his *Begriffsschrift* and later in his subsequent writings, he had in mind no such magnificent and all-embracing aim as setting up the 'alphabet of human thought': his primary goal was relatively humble and modest, namely to contribute to the possibility of articulating mathematical proofs with such precision and clarity that no doubts about their validity could arise. However, as it turned out, *less* may sometimes be *more*: it was precisely the modesty of his goal which enabled him to draw the project of making human language rigorous down from heavenly heights to the realistic earth.

What is a mathematical proof? Basically, it is a means of demonstrating the validity of a mathematical theorem; where a mathematical theorem is a claim that whenever some premises are fulfilled, then a conclusion is also bound to be fulfilled. A mathematical theorem thus claims that a conclusion is a *consequence* of some premises (where the set of premises may, of course, be empty); and a proof is a demonstration of the fact that a statement follows from a list of statements.

How can we demonstrate that something follows from something else? How can we demonstrate that the statement 'Prague is in Europe' follows from the statement 'Prague is in Europe and Tokyo is in Asia'? In fact in no way at all: should somebody doubt *this*, we would probably conclude either that he simply does not know enough English, or that he is insane; in any case that we are not communicating with him. However, what about the fact that the statement '*z* is a number divisible by six' follows from '*x* is a number divisible by two', '*y* is a number divisible by three' and '*z* is the product of *x* and *y*' (i.e. that the product of a number divisible by two with a number divisible by three is divisible by six)? If somebody did not know, or doubted, this, we would not necessarily take him to be a lost case; we might feel that we *could* demonstrate this to him. What we would say to him in such a case would be, roughly, the following: 'That x is divisible by two means that there is a number n such that $x = n\times2$; that y is divisible by three means that there is an m such that $x = m\times3$. But as $z = x\times y$, it is the case that $z = (n\times2)\times(m\times3)$. And this further means that $z = (n\times m)\times(2\times3)$, i.e. that $z = (m\times n)\times6$. However, this is nothing else than that z is divisible by 6.'

What did our demonstration of the validity of the instance of consequence in question consist in? Its point clearly was in decomposing the instance into simpler steps: to show that S is the consequence of S_1, S_2 and S_3 we presented a sequence S_1, S_2, S_3, ... S such that every S_i (for $i = 4, ...n$) is a consequence, and an *obvious* one[2], of S_1, ..., S_{i-1}; and that $S_n = S$. No reasonable being can fail to see both the validity of the individual steps and the fact that they add up to the original instance of consequence to be proved. A typical proof thus consists in the decomposition of an instance of consequence into a chain of obvious instances of consequence – what makes it possible is the fact that some instances of consequence are more obvious than others.

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[1] There were (and indeed there still are) philosophers who believed that formal logic can actuallysolve (or dissolve) *all* philosophical problems (perhaps Rudolf Carnap in his Vienna Circle period). However, I am afraid that this attitude has done the interaction between logic and philosophy more harm than good.

[2] Note that this requirement makes the definition different from the usual formal definition of a proof.

Frege wanted to develop a method enabling us immediately and unambiguously to recognize the validity of a given proof; he wanted to assemble a manual of elementary logical transitions which are capable of constituting the basic steps of proofs – so that a proof could be checked simply by comparing its steps with this manual. (As I have noted elsewhere[3], what he was after was an inventory of acceptable elementary logical transitions analogous to the inventory of reprehensible deeds assembled in the laws.) Doing this he realized that the task could be effectively accomplished only if he made language more precise, in the sense of ridding it of everything which is not substantial from the viewpoint of consequence. In natural language, the same thing can be expressed in many different ways and our talk can be augmented by many things which do not influence the 'judgeable content' – and it is thus necessary to distinguish between matters of mere stylistic variation and the real differences in content.

Notice that in this phase, Frege's efforts are no longer so distant from what Leibniz talked about: we are heading for a language containing only those linguistic means which really do express *content*; those which have other functions and could thus mislead us are dispensed with. However, Frege, unlike Leibniz, now has a method to tell these two kinds of means apart – the criterion is the nontriviality of 'inferential role'. Frege states that 'die Inhalte von zwei Urtheilen in doppelter Weise verschieden sein können: erstens so, dass die Folgerungen, die aus dem einen in Verbindung mit bestimmten andern gezogen werden können, immer auch aus dem zweiten in Verbindung mit denselben andern Urtheilen folgen; zweitens so, dass dies nicht der Fall ist'[4] ([7], p. 2-3). For two judgments S and S', to differ in the former way means that for all judgments S_1, ..., S_{i-1}, S_{i+1}, ..., S_n, S_{n+1} it holds that S_{n+1} follows from S_1, ..., S_{i-1}, S, S_{i+1}, ..., S_n, if and only if it follows from S_1, ..., S_{i-1}, S', S_{i+1}, ..., S_n; whereas the judgments S and S' differ in the latter way iff there exist some S_1, ..., S_{i-1}, S_{i+1}, ..., S_n, S_{n+1} such that S_{n+1} follows from S_1, ..., S_{i-1}, S, S_{i+1}, ..., S_n, but not from S_1, ..., S_{i-1}, S', S_{i+1}, ..., S_n. The 'judgeable content' is thus, for him, precisely that part of content which is shared by any two judgments which differ at most in the first way; and it is merely this part which is to be expressed by the 'concept script'. Thus 'alles was für eine Richtige Schlussfolge nöthig ist, wird voll ausgedrückt; was aber nicht nöthig ist, wird meistens auch nicht angedeutet; *nichts wird dem Errathen überlassen.*'[5] (*ibid*, p. 3) And it is his concentration on inferential behavior that enables Frege to dispense with everything which is not substantial from the viewpoint of consequence and to let the 'substantial backbone' shine – which then leads not only to an effective rendering of a criterion of consequence, but also to the *materialization of the unity of sense within the multiplicity of surface forms*.

Thus, whereas before Frege it seemed that devising a perfect language, assembling a Leibnizian 'alphabet of human thoughts', would presuppose gripping raw ideas or thoughts, comparing them with possible expressive means and choosing the most adequate ones, from Frege's considerations there emerges a quite different methodology[6]. We have to erase all

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[3] See [24], §2.1.

[4] 'The contents of two judgments may differ in two ways: either the consequences derivable from the first, when it is combined with certain other judgments, always follow also from the second, when it is combined with these same judgments, or this is not the case.'

[5] 'Everything necessary for a correct inference is expressed in full, but what is not necessary is generally not indicated; *nothing is left to guesswork*.'

[6] Compare this situation with the situation of someone who wants to check the truth of some statements about the world. What seems to be needed is a comparison of the statements with the raw

differences between expressions which do not influence the expressions' 'inferential properties'. Needless to say that in contrast to the previous one, this methodology is manageable; and thus it constitutes a real breakthrough towards assembling a 'perfect' language.

Hence the brand new period in the development of logic (with its profound influence on philosophy) which Frege's approach initiated is the result of precisely the reasonable 'down-to-earthness' of his original aim and of the ensuing relatively narrow construal of the concept of logic, a construal of logic as basically a matter of a canonization of consequence. I think that this indicates that it is this narrow delimitation of logic which we should stick to.

2. Morals of the Fregean approach

What is distinctive of this conception of logic? Besides the issues discussed in the previous sections, I would like to stress two crucial points: first, according to the Fregean approach, the *primary* subject matter of logic are not *objects and their properties* as constituting the world, but rather *propositions* as constituting the 'logical space' of inferential relationships; second, propositions are to be approached via the sentences which express them.

The substantiality of the move from considering relations among objects (particulars) to those among propositions may not be immediately apparent; but it is, as far as I can see, the very crux of Frege's approach to logic. Frege himself explicitly urges the point in an essay called 'Booles rechnende Logik und die Begriffsschrift' ([8]), in which he compares the merits of his *Begriffsschrift* with those of Boole's *Algebra of Logic*. Frege points out that what is distinctive of his approach (as contrasted to that of Boole, which he considers as a direct continuation of the project of Leibniz), is that he does not proceed from concepts to judgments, but rather the other way around. 'Im Gegensatz zu Boole,' writes Frege (p. 17), 'gehe ich von den Urteilen und deren Inhalten statt von den Begriffen aus. ... Das Bilden der Begriffe lasse ich erst aus den Urteilen hervorgehen.'[7] And he continues (p. 18): 'Statt also das Urteil aus einem Einzeldinge als Subjecte mit einem schon vorher gebildeten Begriffe als Praedicate zusammen zu fügen, lassen wir umgekehrt den beurteilbaren Inhalt zerfallen und gewinnen so den Begriff.'[8,9].

Why did Frege choose this approach? The crucial point seems to be that if we want to explicate concepts independently of judgments, we have no feasible criteria of individuation, and hence no feasible way to grasp concepts as real, demarcated entities. We would be left with conceiving concepts as something mental and to seek criteria of their individuation in

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reality which they claim to capture and to find out whether the two things are in accordance. However, what could 'accordance' between the string of letters 'There are Elephants in Africa' and the African elephants consist in? Surely not in a similarity akin to that which obtains between, say, a thing and its image in a mirror! In this way, the seemingly straightforward correspondence theory of truth gives way to various other accounts of truth which do not assume that we could get hold of a raw reality to be checked for being correctly mirrored by our statements.

[7] 'Unlike Boole, I start out from judgements and their contents, and not from concepts. ... I let the building of concepts proceed from the judgements.'

[8] 'Instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of a possible judgment.'

[9] Cf. [1], §2.II.2.

psychology – which would cause us to erase the boundary between concepts (Begriffe) and mental presentations (Vorstellungen), and this would, in turn, lead us to erasing the boundary between what is really the case and what is thought or imagined to be the case. In contrast to this, we do have a relatively clear criterion of individuation for propositions: propositions are 'judgeable contents' ('beurteilbare Inhalte'), and as such they are 'that which is shared by all sentences not differing by their inferential properties'.

Thus, Frege's strategy appears to be the following: First, we must study and systematize correct inferences. Then we can divide differences between sentences into those which are, and those which are not, inferentially significant. Thereby we establish an equivalence relation between statements ('not to differ significantly from the viewpoint of inference') and we can proceed, via abstraction, to (judgeable) contents of statements – such judgeable contents are what Frege called *thoughts* and what we call *propositions*. And then we can decompose thoughts into unsaturated functions and their saturated arguments, objects (by means of 'subtracting' the objects), thus gaining concepts understood as functions mapping objects onto truth values.

The bite of Frege's conception can be appreciated if we consider, e.g., his quarrel with Husserl about the nature of number concepts[10]. What Husserl proposed was a conception of numbers which was of a piece with his overall phenomenological inclination: a number is, according to him, to be abstracted from our presentations of 'sets, multiplicities and definite objects' ([19], p. 9) and is thus based on 'elementary psychic data' (*ibid*., p. 131). Frege's disagreement with such an approach stemmed not only from his general disagreement with any attempts to base logic and mathematics on facts of psychology (and in fact here Husserl's defense could have been that what he had in mind was *transcendental* psychology), but especially from the fact that Husserl's attitude does not provide for the vital distinction between what really is the case and what somebody takes to be the case, i.e. between 'being true' and 'being taken as true'. For Frege any logical consideration importantly rests on the concept of objective truth; and hence it presupposes entities to which the concept applies, *viz* judgments or thoughts. As he puts it in an overview of his approach to logic: 'Das Eigenartige meiner Auffassung der Logik wird zunächst dadurch kenntlich, dass ich den Inhalt der Wortes 'Wahr' an die Spitze Stelle, und dann dadurch, dass ich den Gedanken sogleich folgen lasse als dasjenige, bei dem Wahrsein überhaupt in Frage kommen kann.'[11] ([10], p. 273) And as he adds in another posthumously published paper: 'Eine Erkenntnis kommt dadurch zustande, dass ein Gedanke als wahr anerkannt wird. ... Als Erkenntnisquelle sehe ich das an, wodurch die Anerkennung der Wahrheit, das Urteil, gerechtfertigt its.' ([11], p. 287)[12].

Thus, what, according to Frege, establishes the objectivity of a concept (e.g., a number concept, like *three*) is the objectivity of the truth values of sentences or judgments containing the corresponding word ('three'). We all agree that a sentence like 'Venus has three moons' has a definite truth value independently of whether anybody thinks it is true or false. (Our agreement is a matter of the fact that our language is governed by certain rules, to which all of

[10] For a detailed discussion of this controversy see [6].

[11] 'What is distinctive about my conception of logic is that I begin by giving pride of place to the content of the word 'true', and then immediately go on to introduce a thought as that to which the question 'Is it true?' is in principle applicable.'

[12] 'When someone comes to know something it is by his recognizing a thought to be true. ... What I regard as a source of knowledge is what justifies the recognition of truth, the judgement.' For a thorough discussion of this point see [34].

us, its speakers, subject ourselves to be able to communicate.) The concept *three* is what remains when we subtract (in modern terminology: 'lambda-abstract') from a thought expressed by such a sentence everything save the part corresponding to 'three'[13].

Note that this strategy is based not only on the acknowledgment of the primacy of the propositional, but also on the assumption that propositions are inseparably connected with sentences which express them – that a thought is always *the sense of a sentence*. The reason appears to be that propositions are individuated in terms of inferences, and inferences are relations between (meaningful) sentences. Hence we can get a grip on propositions via gripping sentences in their inferential relationships; and there is no other way. Thus, Frege sowed the seed of the linguistic turn with its conviction that 'a philosophical account of thought can be attained through a philosophical account of language, and ... that a comprehensive account can only be so attained.' ([4], p. 5).

It may be helpful to elucidate the idiosyncrasy of propositions from a different angle, by briefly discussing Wilfrid Sellars's attack on the doctrine of the traditional empiricism (see [31]). What Sellars pointed out was that the doctrine's apparent acceptability depended upon its proponents ignoring the distinction between particular objects and propositions. The empiricist picture criticized by Sellars is based on the idea that the outer world impinges on a subject producing various sorts of 'sense data', which then constitute the most basic and infallible layer of the subject's knowledge, upon which she may build further, non-immediate layers. Thus, rays of light reflected from a green tree make the subject see that there is a green tree before her, and from this 'perceptual knowledge' she can further infer, e.g., that there is something which is green before her. This picture, Sellars argued, presupposes that there is a causal chain leading from an external source to a sense organ of the subject in question and which is then continued, 'within' the subject, by a justificatory chain leading to the 'less immediate' varieties of knowledge. What Sellars pointed out was, in effect, that causal chains and justificatory chains inhabit different spaces (the first of them belonging to the spatiotemporal world, the other to the 'logical space of reasons') and hence cannot be made continuous with each other. In other words, if we perceive a green tree, then the endpoint of the relevant causal chain is a perception of the green tree (a kind of constellation of something within our eye and/or brain), whereas the starting point of the relevant justificatory chain is the *belief* that there is a green tree. And whereas we can have a perception of a green tree without possessing the concepts *green* and *tree* (thus in this sense not knowing that what we perceive is a tree and is green), we cannot have the belief.

Considerations of this kind made Brandom ([1]) count Sellars as an ally of Frege, and both of them as followers of Kant, in regarding propositions as a crucial *sui generis* which is more basic than that of concepts. Moreover, Brandom argues that it is precisely *propositional knowledge* which is crucially characteristic for rationality of the distinctively human kind. To be rational in this sense is to have beliefs, desires etc. and to act according to them; and beliefs are propositional in nature. As Brandom would put it, *rational* agents are first and foremost those who are able to give and ask for *reasons*, and reasons are what can figure within inferences, i.e propositions. 'Behavior is made intelligible,' Brandom writes (p. 83), 'by exhibiting it as rational, given various beliefs and pro-attitudes, and to do that is to exhibit a piece of practical reasoning that is taken somehow to stand behind or be implicit in the behavior. The imputed reasoning shows why an organism with the states and attitudes that

[13] Frege then argues that in this case what we have to substract is a *concept*, perhaps 'the moon of Venus', so that the entity expressed by 'three' is a *second-order* concept.

provide the premises *ought*, rationally, to behave in the way specified by the conclusion. But *what can serve as a premise in reasoning must have a propositional content*.'

Objects are, in the typical case, entities which we can see, touch, or kick (in short causally interact with); propositions are those which we can know, deny or infer from each other. Thus whereas the *modus vivendi* of (spatiotemporal) objects is causal interaction, the *modus vivendi* of propositions are logical relationships – nothing can be a proposition unless it can be negated, conjoined with other propositions, etc[14]. Propositions are creatures and vehicles of *reasoning* and hence should be in the primary focus of logic. And the way to get a grip on them is to investigate the overt tools of reasoning, statements – propositions are what emerges when we rid statements of everything irrelevant from the viewpoint of inference.

3. The Principal Alternative: The 'Russellian' Notion of Logic

The most common alternative to this understanding of logic is the conception according to which logic captures the most general traits of the world, especially the boundaries of what is possible within the world. According to this conception, the truths of logic are about the world and about the objects to be encountered within the world in the same way in which the truths of natural science are – the only substantial difference being a matter of their generality. This means that unless we want to see the truths of logic as contingent, we have to assume that truths about the world which are spelled out by logic are, in contrast to those spelled out by natural science, somehow so general that they are no longer contingent, but become necessary. This is the conception of logic put forward, within the post-Fregean context, most illustriously by Bertrand Russell: '[L]ogic is concerned with the real world just as truly as zoology, though with its more abstract and general features' ([30], 169-70).

What can be said about the object/proposition dichotomy from this viewpoint? Well, for Russell, there are really no propositions in our sense (he uses the term 'proposition' for statements, the linguistic entities). The only entity which is semantically relevant for a statement is a certain *fact*: the statement is true if the fact is present, whereas it is false if it is absent. Facts are kinds of conglomerates of objects which, 'just as much as particular chairs and tables, are part of the real world' ([29], p. 42). Thus, the fact that a tree is green appears to be a specific kind of complex object, somehow consisting of the tree and greenness. Russell (*ibid.*, p. 80) claims: '... I should always wish to be engaged in the study of some actual fact or set of facts, and it seems to me that that is so in logic just as much as in zoology'. The idea behind such a construal of logic is that whereas a statement like 'The king of France is bald' may be true or false depending on the actual status of a relevant part of the world (i.e. on the presence, respectively absence, of the fact made up of the king of France and baldness), when we move to more general statements we ultimately reach those which are so general that they no longer concern only a specific part of the world, but somehow the world as a whole, and thereby they lose their contingency. The example of such a statement is 'If one class is part of another, the term which is the member of the one is also a member of the other'. Russell (*ibid*., p. 43) describes the situation as follows: 'There are facts concerning particular things or particular qualities or relations, and, apart from them, the completely general facts of the sort that you have in logic, where there is no mention of any constituent

[14] See [27].

whatever of the actual world, no mention of any particular thing or particular quality or particular relation, indeed strictly you may say no mention of anything'.

Since the establishment of model theory in the modern sense, this approach to logic has tended to be slightly mutated (reflecting the shift from what van Heijenoort calls 'logic as language' to what he calls 'logic as calculus'[15]). According to this modified version of the conception, logic reports to us what holds in every member of a certain class of formal structures, which represent all and only possible states of our world (see [5]). This seems to render Russell's 'more abstract and general' as 'universally valid'. That this is a notion of logic taken as virtually self-evident during recent decades (especially among mathematically-minded logicians) is documented by the fact that Kreisel's replacement of the problem of the relationship between logical validity in the intuitive sense and 'model-theoretic validity' with the problem of the relationship between the truth w.r.t. all possible model structures and the truth w.r.t. the restricted class model structures worked with by model theory[16] has been almost universally accepted as an unproblematic move.

I think that this way of understanding logic, although it might seem *prima facie* plausible, is in fact plainly untenable, as it stands, for the simple reason popularised long ago by David Hume. The reason is that we can report what is the case, but not what *must be the case*, nor what *cannot fail to be the case*. These are simply not the kinds of things which can be *reported* – however many times we see something happen or be the case, we cannot be sure that it is *bound* to happen or is *necessarily* the case. So the idea that proceeding from the usual, observable facts towards ever greater abstractness and generality will ultimately lead us to some kind of facts whose superior abstractness and generality secure for them necessity and inevitability is not really plausible.

Transposed into the modern, 'model-theoretical' setting, the Humean line of thought suggests that we can never find out, by observing the world, whether a proposed class of model structures really represents all its possible states. If Sher ([33], p. 139) notes that 'Tarski has never shown that the set-theoretic structures that make up models constitute adequate representations of all (formally) possible states of affairs', then our point is that the only way to show this would be to show that the class of structures does justice to what is logically true – and consequently that explaining logical truth as that which holds in all of them would be circular[17].

To make this point obvious, suppose somebody asks how we know that a statement, say, $\neg(P(a)\&\neg P(a))$, is logically true. Surely our answer cannot be 'It holds in all (the model structures capturing) the possible states of the world – I have gone through them all and have not encountered a single one in which it would *not* hold'; it would have to be something like 'a thing simply cannot be P and simultaneously not-P', or perhaps '["]to be P and not-P["] makes no intelligible sense'. This indicates that not 'a logical truth is true *because* it is valid in all possible structures', but rather '*because* something is a logical truth, there cannot be a structure in which it does not hold'. (It is true that we can sometimes *discover* that something holds in all structures of a certain class – but unless the class is finite, we can hardly do so by

[15] See [15].

[16] See [22]. The difference, in Kreisel's view, lies in the fact that among the model structures with which model theory normally works there is, for example, no one with the whole set-theoretical universe as its carrier.

[17] Cf. [24], Chapter 4.

going through all the structures; we have to somehow deduce it from some properties constitutive of the very class[18].)

This indicates that the 'Russellian' notion of logic, if it is to be minimally feasible, has to be modified in the sense that the truths of logic are somehow a matter of what is 'within us', of what is somehow 'imposed' on the world by us. This leads to the well-known Kantian response to the Humean challenge: We can know what is necessary within the world because the necessities somehow stem from us, are somehow a matter of 'the structure of our epistemological apparatus'. This permits us to save a part of the original intuition constitutive of the 'Russellian' conception of logic; however, it also demands its significant modification.

The modified conception claims that logic reports the most general traits of the world *as we think it*. However, this clearly cannot be taken to imply that logic is simply a matter of studying our actual thinking (or cognition) – the case against all kinds of psychological construals of logic was made so vehemently by Frege and by many others since, that there is, I hope, no reason to repeat it here (cf. the previous section). Hence again, to make this construal of logic feasible, we must accept that what it investigates somehow 'transcends' our actual thinking, that it is a matter of the boundaries of what we *can* and what we *cannot* think.

However, even this conception of logic may still be subject to serious objections. The principal one is, I think, that which can be found sketched in Wittgenstein's *Tractatus*: it is the objection that we cannot think about (let alone form a theory of) what we cannot think[19]. How can we see something as a boundary without being able to conceive of the outside of the boundary? It seems that with this conception of logic we would need to see the boundaries of thinking as something which we *can*, but at the same time *cannot* surpass[20].

I think that it is the concept of *rule* which helps us overcome this difficulty (and this is also the reason why the concept plays such a crucial role within Wittgenstein's later philosophy). A rule is something which *can* be, but at the same time *ought not* to be, violated. A rule draws a boundary, but not one which is utterly unsurpassable: the boundary drawn by a rule *can* be surpassed – even though at some cost. We can violate the rules of chess – at the cost of ceasing to play chess.

However, the straightforward embedding of the concept of rule into the modified 'Russellian' conception of logic still yields a picture which is questionable: namely to the notion of logic as the theory of the rules of correct thinking, i.e. of how we ought to think. This notion takes logic to be a kind of specification of what is to be happening within our heads if we are to 'think correctly'. However, I think that the Fregean arguments against the psychological construal of logic extend even to this case: logic is not about what is going on

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[18] Notice that this is not an argument against the 'platonistic' construal of mathematics. Even if we granted that facts about a mathematical structure of the kind of those employed by model theory can be reported using an 'inner eye' in the same way in which we report facts about the empirical world using the real eye, it would still not follow that it would be possible to report what holds in every member of an infinite class of such structures.

[19] See Wittgenstein ([35], §5.61): 'Was wir nicht denken können, das können wir nicht denken; wir können also auch nicht sagen, was wir nicht denken können.' ['What we cannot think, that we cannot think: we cannot therefore say what we cannot think.']

[20] Somebody could try to overcome this difficulty by appealing to some kind of 'metalevel thinking' from which we somehow can cross the boundaries not crossable within the 'object-level thinking'. (Cf. Kleene's, [21], pp. 2-3, proposal that 'we simply put the logic which we are studying into one compartment, and the logic we are using to study it in another'.) But this is obviously futile: if we *are* able to cross them in any sense at all, they are clearly not the real boundaries of our thinking.

in anybody's head, not even in this normative sense, i.e. as being a recipe. The reason is, again, that logic is concerned with what is true (it is, of course, directly concerned only with the noncontingent side of what is true, i.e. with *necessary* truth and with inference), which does not directly depend on any goings-on within a head[21]. That 'Someone is bald' is true if 'The king of France is bald' is, is an objective fact independent of the fact of how a real person moves from the knowledge of the truth of the former to that of the truth of the latter.

However, at this point the dismantling of the 'Russellian' conception appears to be completed – there seems to be nothing more left of it. If what we have claimed so far is right, then logic is best seen, just like Frege urged, as the study of objective inferential relationships between propositions, which result from the rules which govern our way of handling sentences which express them. Viewed thus, logic is the systematic study and 'canonization' of inferential rules which are constitutive of the core of our language.

4. Logic as a Matter of Inference Rules

However, is the upshot of the previous section that logic is a specific part of linguistics? Does it mean that logical studies are in fact peculiar grammatical studies? Surely not: there is, of course, an important sense in which logic is non-empirical and normative.

The first thing to realize is that, as we have stated, what logic canonizes are *rules*. This means that it does not merely spell out *regularities* of the way people use language; what it spells out are, as Brandom ([1]) puts it, *proprieties*[22]. And logic does not put them forward as mere 'linguistic' reports of what is held to be correct ('The speakers of English take it to be correct to infer "Someone is bald" from "The king of France is bald"'), it puts them forward as claims with genuine normative import ('It is correct – for us speakers of English – to infer "Someone is bald" from "The king of France is bald"'.)

Moreover, logic deals with the core of the inferential structure of language, which is its target, as with a rigid, unchanging structure. This is what makes it possible to apply mathematics to logical investigations, to have a 'mathematical logic'. Thus we can say that logic addresses a 'mathematical' structure which is – which *happens to be*, we can say – embodied – imperfectly, as the case may be – by the inferential structure of the core of our language. (This is, in itself, no specialty of logic – consider, e.g., geometers studying

[21] It is, of course, indirectly dependent on 'what is going on in heads' in the sense that if nothing went on in any head, there would be no thoughts and no sentences to be true, and hence there would be, in this sense, no truth. Therefore Frege ([9], §26) says: 'So verstehe ich unter Objektivität eine Unabhängigkeit von unserem Empfinden, Anschauen und Vorstellen, von dem Entwerfen innerer Bilder aus den Erinnerungen früherer Empfindungen, aber nicht eine Unabhängigkeit von der Vernunft.' ['It is in this way that I understand objective to mean what is independent of our sensations, intuition and imagination, and of all construction of mental pictures out of memories or earlier sensations, but not what is independent of the reason.']

[22] Of course this presupposes a theory which renders the rules of human linguistic conduct as more than acknowledged regularities – but such theory indeed can be found in Brandom's book. We should also note that the proprieties in questions are again ultimately based on some regularities, *viz* on regularities of what Brandom calls *normative attitudes*, i.e. of the 'takings as (in)correct' of the linguistic behaviour of one's fellow speakers and herself.

geometrical forms 'imperfectly embodied' by things of our world) [23]. In this way it produces claims about this structure which are mathematically certain – but these claims are to be understood not directly about the language, but rather about a formal prism which is taken to be a 'reasonable reconstruction' of the language and which is indeed used as an 'ideal norm'[24].

What is important is that the kind of structure which is studied by logic appears to be so essential for our language that it would not make sense to use the term 'language' for anything which lacks it. (In particular, the structure appears to have to be embodied by anything capable of serving as a means of communication and 'information exchange'.) Note that this is nothing else than what characterizes other words of our vocabulary: we would not call a 'car' or a 'crocodile' anything which does not display the most essential features characteristic of those things for which the names were introduced. This observation also yields an answer to the question of whether on this construal of logic we are perhaps bound to have merely a *logic of English*, a *logic of German* etc., and no *logic simpliciter*. Our logic expresses the normative structure constitutive of *our* language, but thereby the one *which is bound to be embodied by all languages worth the name*. We simply use the term 'language' for certain kind of entities; and one of the criteria of calling something 'language' is that it shares the basic structure we know from *our* language – similarly as the criterion of calling an alien entity a 'car' or a 'crocodile' is that it is close enough to our cars or our crocodiles. This means that the fact that all languages must share a basic logical structure is not a fact of metaphysics, but rather a fact of semantics, concerning the meaning of the word 'language'[25].

To elucidate the consequences of our proposal, let us discuss the claim of Nagel ([23], p. 38-39) to the effect that logic cannot be extracted from the grammar of our language: 'To the extent that linguistic practices display principles of reasoning or show us, for example, something about the nature of arithmetical propositions, it is not because logic is grammar but because grammar obeys logic. No "language" in which modus ponens was not a valid inference or identity was not transitive could be used to express thoughts at all.' Does our construal of logic result into the claim that 'logic is grammar'?

To enable us to answer this question, let us first consider Nagel's latter claim. What does it mean that 'modus ponens is a valid inference in a language'? Is modus ponens a valid inference in English? To answer this question, we have to specify what is to be understood as *implication* in English, for modus ponens tells us that an implication together with the antecedent of the implication entail the consequent of the implication. Could we simply identify implications with sentences of the shape *if A, then B*? It seems that if we did so, then we would have to admit that modus ponens in fact is *not* valid in English – for there are

[23] This account of the role of mathematics in logic coheres, in my view, with the overall characterization of the role of mathematics within our coping with the world as put forward by Kitcher ([20]).

[24] See [26]. In this paper I also urge that it is futile to conceive of logic as being about a non-empirical reality – as being the inquiry into a world of eternal propositions independent of the fact whether these propositions happen to be expressed by sentences of a language. I argue that the value of studying any 'pure' mathematical structure is in that it is a structure of a thing with which we have to do within our world; and especially that studying inferential relations within a structure of abstract propositions makes (extra-mathematical) sense only if this structure can be used as a 'prism' to look at our real language and our real argumentation.

[25] It is a consideration of this kind that appears to have led Davidson ([2]) to the conclusion that the very idea of a language untranslatable into ours is incoherent.

surely many cases in which a sentence B is *not* entailed by *if A, then B.* (Consider. e.g., the cases where the consequent of the *if ... then* sentence is not a self-contained sentence, like *If a farmer owns a donkey, he beats it.*[26])

Moreover, it is unclear what 'a language in which modus ponens is not a valid inference' would amount to. Would it be a language containing implication not governed by modus ponens? But then why would we call the connective in question 'implication' in the first place? Suppose somebody argued that modus ponens is not valid in English and tried to justify the claim by pointing out that the sentences 'Paris is in France', 'Paris is in France or Paris is in China' are true, but the sentence 'Paris is in China' is false. We would surely protest that 'or' is *not* an implication. However, how else could we justify our protest save by pointing out that the inferential behavior of 'or' is different from that of implication – *viz* that 'or' does not obey modus ponens (and other inferential rules constitutive of implication)?

So the concept of 'an implication not obeying modus ponens' is problematic in itself. Moreover, it follows from our considerations that it may be, more generally, problematic to think about a language not obeying our basic logical principles (such as modus ponens). The point is, as we have seen, not that 'such "language" could not be used to express thoughts at all', but that it would be not clear whether we should call such an entity 'language'. (And this is not a metaphysical pronouncement about the essence of language, but merely a semantic gloss on how we (happen to) use the term 'language'.)

Now if this is right, then Nagel's crucial verdict, namely that 'linguistic practices display principles of reasoning ... not because logic is grammar but because grammar obeys logic', is really not intelligible. From our viewpoint, the question *Is logic grammar or does rather grammar obey logic?* is simply a bad question – bad in a way analogous to the badness of the question *does our world obey geometry or is geometry the (idealized) structure of our world*?. The principles we recognize as logical are, as a matter of fact, embodied in our language(s) (although not quite directly, but in the sense that the language(s) can be seen as their imperfect embodiment(s)). It is also true that any language must so embody these principles – for otherwise we would not call it 'language' and we would not call its rules 'grammar'. In this sense 'grammar obeys logic'. However, the rules of logic are idealized versions of grammatical rules (they regulate what follows from what), and so *in this sense*, 'logic is grammar'.

5. The 'Formality' of Logic

There may seem to be one more source of the idea that logic is 'transcendent' to the rules of our language which we have not yet tackled, namely what is usually called the *formality* of logic. Logic is not concerned with the fact that it is correct to infer '*X* is an animal' from '*X* is a dog' or '*X* is a number' from '*X* is a prime'. Such *material* inferences, so the story goes, are a matter of the *content* of our language (in this particular case of the content of the words 'dog', 'animal', 'prime', 'number'); and hence in this sense *are* a matter of the 'grammar' (if we construe the term so as to comprise semantics) of our language. However, the laws of

[26] It might seem that this problem could be dispensed with by some easy gerrymandering: that all that need be stipulated is that *If A, then B* is an implication (proper) only if both *A* and *B* express self-contained statements. However, what exactly is *self-contained*? Is the sentence 'Clinton is the president of the USA' self-contained? There surely exists more than one Clinton!

logic are purely *formal*, they have nothing to do with content and hence nothing to do with 'grammar'.

I think this argument is utterly misguided. What is the *form* of an expression? Given the normal meaning of the term 'form', the form of an expression is that which remains if we abstract from *all* particular expressions (and indeed, we use 'form' in this sense when we speak, e.g. about the 'subject-predicate form' of a statement). However, it is clear that in fact *all* inferences are valid (also) in virtue of the *meanings* of some expressions involved – none of them is valid purely in virtue of the form (no statement may be a logical truth on the basis of, e.g., the fact that it has a subject-predicate form, or that it consists of two sentences linked together by a connective). *Logical* inferences are valid in virtue of the meanings of the expressions like 'and', 'or', 'something' etc., others in virtue of the meaning of such expressions as 'animal', 'dog' etc.

On our construal then, logic is formal for it deals with only those norms of our language which cut across all varieties of our (rational) discourse, for they concern the expressions which can be called 'topic-neutral'. (We can indeed study inferences which are a matter of expressions pertaining to various specific domains, like the inference of '*X* is a animal' from '*X* is a dog'; but the term 'logic' has been simply reserved for the study of the most general ones.) Now the fact is that these norms are presupposed by any kind of discourse which is *about* something – thus logic, as Quine ([28], p. 52) puts it, 'has no objects of its own' (in the sense in which arithmetic has natural numbers and zoology has animals) and *in this sense* it expresses a form, not a content[27].

This way of understanding the formality of logic underscores the contrast between the construal of logic proposed here and the 'Russellian' construals according to which the laws of logic report some very general facts about the world. Let us consider the argumentation Hintikka ([18]) uses to forward his logical system, the so-called 'independence-friendly logic'. What Hintikka claims is that quantified formulas of predicate logic spells out, in effect, the ways we can choose elements from the universe: thus e.g. the formula $\forall x \exists y \mathrm{R}(x,y)$ states that for every *x* we can choose a *y* which stands in the relation R to it. Hintikka concludes that any formula is in fact a codification of a game, the moves of which consist in choosing of individuals from the universe. Given this, he asks: is there a sound reason to restrict the games codified to only those games which are expressible by the formulas of the standard predicate calculus? Why accept only games with full information (i.e. those in which all the information about previous moves is available), why exclude other kinds of games; hence why accept only linearly ordered quantifiers and not branching ones[28]? And as he does not see any such sound reason, he sets up his 'independence-friendly logic' which he views as releasing real logic from the unwarranted chains of linearity.

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[27] Russell ([30], p. 201) famously claimed that 'there are words which express form'. I think that without further ado this is a bizarre thing to say – I think that words which express form in the normal sense of the word would be not 'and' or 'every', but rather 'mould', or indeed, 'form'. And I think that the only possible way to give such a claim an intelligible sense is to say that 'to express form' is to be understood as 'to belong to that stratum of our language which is necessiatated by any talk about any objects'.

[28] An example of a formula with branching quantifiers would be the formula

$$\begin{matrix} \forall x \exists y \\ \qquad\quad \mathrm{R}(x,y,u,v), \\ \forall u \exists v \end{matrix}$$

which is to be interpreted as claiming that for every *x* there is a *y*, and *independently of it* for every *u* there is a *v* so that R(*x*,*y*,*u*,*v*). This is provably expressible by no standard first-order formula.

Hintikka's argumentation is indeed persuasive, however, only *provided we agree that the standard logic is about choosing elements from the universe*. In this case it seems indeed peculiar that logic is not capable of expressing such things as two mutually independent choices – and the laws of traditional logic would appear as perhaps a haphazard selection of principles characteristic of choosing. However, if we opt instead for the conception of logic which we urge here and which we ascribe to Frege, the situation changes: for those basic laws of logic which were articulated by Frege and which have survived, without substantial changes, to the present, do apear to be the most primitive and elementary instances of inference (and hence possible moves in argumentations and proofs).

6. A Case Study: Frege vs. Dipert

The fact that different people understand different things by 'logic' of course causes misunderstandings. In particular, those who subscribe to the 'Russellian' notion of logic and who thus expect it to yield a 'super-theory of the universe' are bound to be disappointed by the results logic in fact achieves. The point is not only that the tasks they expect logic to solve are not acknowledged by those logicians who think about logic in the 'Fregean' way, but, more fundamentally, that these tasks are such that logic simply *cannot* live up to them. As an example of a criticism of the current status of logic based on such a misapprehension, let us discuss the recent paper of Randall R. Dipert ([3]), which diagnoses an overall failure of logic and as a remedy proposes the replacement of logic with graph theory. The crux of Dipert's argumentation is that logic simply cannot underpin an adequate general theory of the world – for no less than six decisive reasons. However it is not diffecult to see that measured by the Fregean conception of logic his criticism turns out to be misplaced:

1. 'Aural and visual structures ... do not seem to be in a logical form, even if they can be wrestled into it.' (*ibid*., p. 333) Indeed – for *logical form* is a form of *propositions* (Fregean *thoughts*), not of things, like perceptions. Logic, of course, can be used as a framework for constructing theories of structures of things (by introducing new, extralogical constants and new, extralogical axioms); but it would still remain essential to distinguish between the structure of a proposition of such a theory and the structure of a thing described by the proposition.

2. 'Logical structure is historically associated with highly conceptualized thought and in fact with thoughts that are easily expressed in natural language.' (*ibid*., p. 334). Again, it is Frege's achievement to differentiate the sense of 'thought' which is interesting for logic from possible broader senses, in which 'thoughts' are a matter for psychology. As Frege tried to show, to trespass into the realm of psychology is fatal for logic.

3. 'Current logical notations, as linear sequences of symbols or 'strings', seem to be irredeemably awkward, or even inadequate, at representing certain quantificational phenomena.' (*ibid*., p. 334) This clearly echoes the argument of Hintikka discussed in the previous section; but as we have concluded, this argument is inconclusive. The point is that it is not clear that the inadequacy of logic to express the quantificational phenomena in question must be seen as a deep shortcoming – for, as we have argued above, it is at least dubious whether the phenomena do belong to the 'logical backbone' of our language and hence whether it is a basic duty of logic to provide for their direct representation. (This is of course not to say that the choice of the particular inferential patterns which Frege elevated to the cornerstones of our logic would be immune to criticism; it is even not to say that the

Hintikkian line of cricicism would be utterly misplaced. It is only to say that it does not in itself provide for a refutation of the Fregean approach to logic.)

4. 'The notion of a logical individual ... has been enormously problematic.' (*ibid*., p. 335) Again, I am afraid that from the Fregean angle there is no such notion: logic simply assumes that there are individuals (in force of there being grammatical subjects), but is (or should be) absolutely neutral to any considerations about the nature of such individuals. (And again, we should not mistake logic for individual substantial theories constructed on top of logic, or within the framework it establishes.) As we have said, logic is characterized by the fact that there are no objects which it could take as its own.

5. 'Predicate logic does not answer, or even frame the question, for example, of *which* one-place properties are basic, and which reducible, or which (if any) two-place relations are basic, and so on.' (*ibid*., p. 335-336). Indeed; and once we drop the idea that logic should spell out properties or structures of things, we can see no reason why it *should* answer or frame such questions. (We must beware mistaking logic for the Russellian, essentially philosophical, doctrine of logical atomism, which was, in effect, a pursuit of the ultimate atoms of the world. As Hacking, [14], p. 315, puts it, 'logic ... should postulate points of convergence or condensation, not atoms.' See also [25].)

6. 'Logic gives nonperspicuous accounts of large and important structural features in the world – the organization of the planetary system or of a Ludwig Beethoven symphony.' Once more, logic in itself does not (or should not) do anything like giving accounts of this kind; but here the author probably does not mean logic itself, but any theory couched in the formal means offered by logic. However, again we have to make the crucial distinctions between *things* (which can be *named* and *depicted*), and *propositions* (which can be *expressed* by a statement). That a detailed propositional description of a Beethoven symphony would consist of a large number of atomic propositions and hence would probably be a long conjunction does not seem embarrassing – unless we confuse the description for a picture or a diagram.

In summary then, logic, in the Fregean sense of the word which we urge to be the most promising, is not meant to solve the tasks Dipert claims it has failed to solve. However is there anything nontrivial now left for our logic to solve? Have we not reduced it to a mere triviality which really has to do with *nothing*? Surely not: as pointed out above, the reason why logic appears to be 'about nothing' is that it is prior to any 'about something', that it articulates those basic structures of our reasoning which enable it to be 'about something' in the first place. And I think this articulation *is* something to achieve.

Besides this, it is essential to distinguish between two ways of seeing the world, in a sense engendered by the object/proposition dichotomy urged above. Viewed from one angle, the world is the world of objects in various ways (causally) connected one to another (by means of the various relations which are the subject matter of science). However, seen from another angle, it consists of facts, and as facts are nothing other than *true propositions*, they are interconnected by *logical* relations, especially by the relations of entailing, of being a reason for. Thus, on this construal, facts are not simply complex objects belonging to the same world as objects proper: the 'world of objects' and the 'world of facts' are alternative ways to grasp the same world. Now logic has very little to do with the world seen in the first of these ways – it only prepares (universal) framework for ('scientific') theories which do deal with it. It has far more to do with the world seen in the second way; for it spells out *its* structure. Thus, if we want to see logic as a theory of the world, then we have to see it as a theory of the world in *this* sense: it can be reasonably seen as revealing the structure only of entities of a very specific sort, *viz* propositions. To compare logical formulas with paintings,

graphs or schemes is like criticizing a hammer on the score that it is not good for extracting aching teeth[29].

6. Conclusion

We claim that it is reasonable to construe the term *logic* relatively narrowly, as a theory of (correct) inference (which primarily follows the aim of 'inventarization' and 'canonization' of the most general and the most elementary steps in inferences). Inference, then, is at best considered as a relationship between sentences (and only via this as a relation between 'thoughts' or propositions) – as a relationship which is a matter of the most general norms and rules constitutive of our language (and which are necessarily shared by everything which we would be willing to call 'language'). We have indicated that this construal of logic is congenial with the approach pioneered by Gottlob Frege.

It can be objected that the difference between Frege and Russell has been exaggerated – indeed, Russell, in many respects, is himself surely a smooth continuator of Frege. This is true; and I am not claiming that there is a grave difference between the logical *praxis* of the two scholars. However, when it comes to the way they *understand* the praxis, the difference, as I have tried to show, is no longer insignificant; and what is important, amplified through the works of their followers it may become a true source of misunderstandings and misapprehensions.

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